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## ABSTRACT

The Minnesota School Mathematics and Science Teaching (MINNEMAST) Project is characterized by its emphasis on the coordination of mathematics and science in the elementary school curriculum. Units are planned to provide children with activities in which they learn various concepts from both subject areas. Each subject is used to support and reinforce the other where appropriate, with common techniques and concepts being sought and exploited. Content is presented in story fashion. The stories serve to introduce concepts and lead to activities. Imbedded in the pictures that accompany the stories are examples of the concepts presented. This booklet contains a unit on numbers and set concepts. The topics include representing numbers by constructing or describing equivalent sets, using words to represent numbers and using special written symbols to represent numbers. Some of the activities are designed for using the "Minnebars," in which the length of a bar represents a number. Although the formal introduction of addition does not occur until Unit 7, the Minnebar activities serve as pre-addition exercises. Union and intersection of sets are also introduced in this unit. Worksheets and commentaries to the teacher are provided and additional activities are suggested. (JP)

ED 094985

**MINNE**

**Math**

**And**

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**UNIT IV**

**SETS, NUMBERS,  
NUMERALS**

■ MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER

1965

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MATHEMATICS  
FOR THE  
ELEMENTARY SCHOOL

UNIT IV

Sets, Numbers, Numerals

11

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★ Starring indicates content which is particularly important to the sequential development or evaluation of the program. We ask that all participating teachers try this starred material. It is expected that much of the remaining material will also be used; how much will depend on individual class needs and time available.

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## INTRODUCTION

In Unit II, sets were compared using one-to-one correspondence. In that unit, the sets to be compared were simultaneously available to the child.

It is often desirable to compare sets which are too greatly separated in space or time to be compared directly. Ugboo's big problem was to compare the set of sheep in the morning to the set of sheep in the evening.

To solve the problem of comparing sets which cannot be compared directly, a symbol is used to represent the number of members of one set. Any symbol for a number is called a numeral. Ugboo constructed a set of stones for a numeral. An answer to the question, "How many?" is a numeral. This unit is about numerals.

Rather than constructing a new equivalent set, an already existing, familiar equivalent set may be referred to, as, "I have as many apples as there are legs on a horse."

The Minnebars suggest a slightly different way of representing number. Here the length of a bar represents a number. The activities with the Minnebars involve the children in matching exercises. In this case, however, the term "matching" is used in a slightly different way. In earlier units the children were asked to match the members of two sets to determine which set had more members than the other. In the matching exercises in this unit, the children are given a set of units (in Minnebars) and are asked to construct a second set of units (in Minnebars) which will be equivalent to the first set. It is possible that the children will transfer what they have learned about one-to-one correspondence to these activities. However, there is no insistence that they do this, since they can play and benefit from the games without this transfer. Although the formal introduction of addition does not occur until Unit VIII, the Minnebar activities also serve as pre-addition exercises.

It is recommended that the children be allowed free play with the Minne-bars several days before starting the activities given on page 22.

The story, "Tal's Aching Back", suggests how the practice of using sets of stones as symbols for numbers may have evolved into counting, or using words to represent numbers. Convenience is probably one reason for such a change. Tal found it easier to carry a series of words in his head than to carry a bag of stones in order to symbolize a specific number. By repeating the words in the same order each time he used a set as a symbol for a number, Tal found that he needed only to say the last word in the series to communicate the number he wanted. It was easier to say "I have four chairs," than to say, "I have one, two, three, four chairs." The following activities are designed to help the children learn the series of words used in the story to represent numbers.

The story, "Ruth Ann at the U.N.", broadens the ideas expressed in "Tal's Aching Back" by showing that other countries use different series of words to represent numbers, even though many of them use the same written symbols.

"The Birthday Party" serves as a good review of numeral recognition.

Union and intersection of sets is included at the end of this unit. Through the game, "Sets Up", the children are led to discover important facts about the number properties involved in the union and intersection of sets.

Introduce the story about Ugboo with the following sentence:

"One night when David was ready for bed, his father read him this story:"

# UGBOO'S

## Big

### Problem

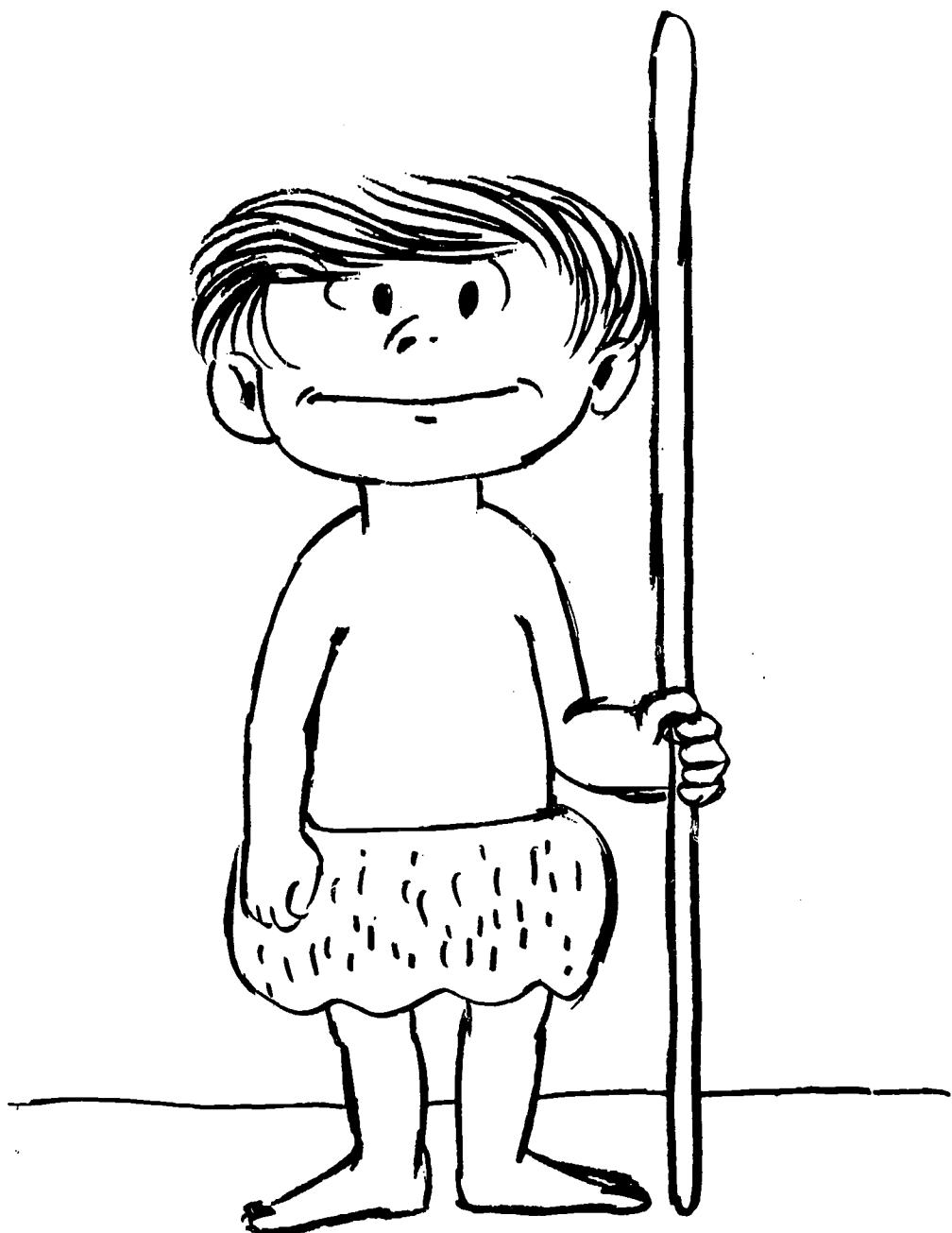
by

PAUL C. ROSENBOOM



Representing Numbers Using One-to-One Correspondence

## UGBOO'S BIG PROBLEM



Here is Ugboo. He lived a very long time ago.

## UGBOO'S BIG PROBLEM

Here is Ugboo. He lived a long, long time ago. This is the way he counted: "One, two, many". He did not know any other way to count more than two. At that time no one knew how to count more than two.

Ugboo had a job. He watched sheep. He watched the sheep of the whole tribe. Every morning he would let them out of the fold. He would take them to a grassy place. Ugboo had to watch the sheep while they ate grass. He had to keep his eyes open because the lions and the wolves liked to eat sheep. They would steal sheep and eat them if Ugboo wasn't looking. At night Ugboo would take the sheep home and let them into the fold.

One evening when Ugboo came home with the sheep, the chief whose name was Zarathustra, met him. (In those days the chief of a tribe always had a long name to show that he was important.) Zarathustra looked at the sheep. He said, "Are these all the sheep of the tribe? Don't you have more?"

UGBOO'S BIG PROBLEM



Chief Zarathustra was angry with Ugboo.

## UGBOO'S BIG PROBLEM

Ugboo answered, "These are all the sheep of the tribe. They are the same sheep we had this morning."

Zarathustra said, "They don't look like as many. Did you fall asleep today while you were supposed to be watching them? I think you did."

Ugboo began to cry. "I did not fall asleep. I kept my eyes open all the time."

Zarathustra was angry and shouted, "I think you fell asleep and a wolf or a lion stole some sheep. It looks as though there are fewer sheep than there were before. If you lose any more sheep, I will punish you. The tribe needs the sheep for wool, skin, and meat." Then the chief went away.

Ugboo was very sad. He said to himself, "How can I be sure that there are as many sheep now as there were before? How can I show big chief Zarathustra that I did not lose any sheep? There were many sheep before and there are many sheep now. How can I or anyone else tell the difference between many sheep and many sheep?"

Ugboo was afraid of Zarathustra. The chief was the biggest and strongest man in the tribe. All the others were afraid of him, too. Ugboo did not want Zarathustra to punish him.

Ugboo thought and thought. He thought all evening about his problem. At supper his mother said, "Ugboo, why are you so quiet? On other nights you chatter all the time like a monkey."

He replied, "I am thinking."

His mother said, "Don't think too much. You might get a headache. Thinking is hard work."

## UGBOO'S BIG PROBLEM



Ugboo cut the meat and said, "One piece for Nip, and one for Snip,  
and one for Snap, and one for little Norum."

## UGBOO'S BIG PROBLEM

Ugboo told his mother about his problem. She said, "You are wasting your time. Everybody says 'one, two, many' and that's all there is. More than two is many. Now stop all this nonsense and help me serve supper. Here's a stone knife and a big piece of deer meat. Cut off a small piece for each of your brothers and sisters."

Ugboo did as his mother told him. He began cutting and said at the same time, "One piece for my brother Nip, and one for my sister Snip, and one for my other sister Snap, and one for little Norum." Then he carried the pieces to the other children and gave a piece to each one. He gave himself a piece, too.

While they were eating, Ugboo said to himself, "One piece for each child, one child for each piece of meat. No children left over. No pieces left over. There are just as many pieces as children and just as many children as pieces of meat."

Suddenly he felt as though a light had lit up in his head. He felt as though he had been in the dark until now.

He jumped up and yelled, "I've got it! I've got it!"

His mother asked, "What have you got? A stomachache?"

Ugboo said happily, "I know how to find out whether there are fewer sheep at night than in the morning. I've got it! I've got it!"

How do you think Ugboo found the answer to his problem?

Let the children make suggestions.

UGBOO'S BIG PROBLEM



Two sheep out of the fold - two stones in Ugboo's sheepskin bag.

## UGBOO'S BIG PROBLEM

As soon as he got up the next morning, Ugboo went to the edge of the lake near his cave. He took with him a sheepskin bag which his mother had made for him. He began looking for little stones and pebbles. He picked up every one he found, and put it in his bag. When the bag was full, he threw it over his shoulder and carried it to the sheep fold. After he emptied the bag on the ground, he let the sheep out of the fold, one at a time.

As each sheep left the fold, Ugboo put a pebble in the bag. Then he led the sheep to the big chief, Zarathustra.

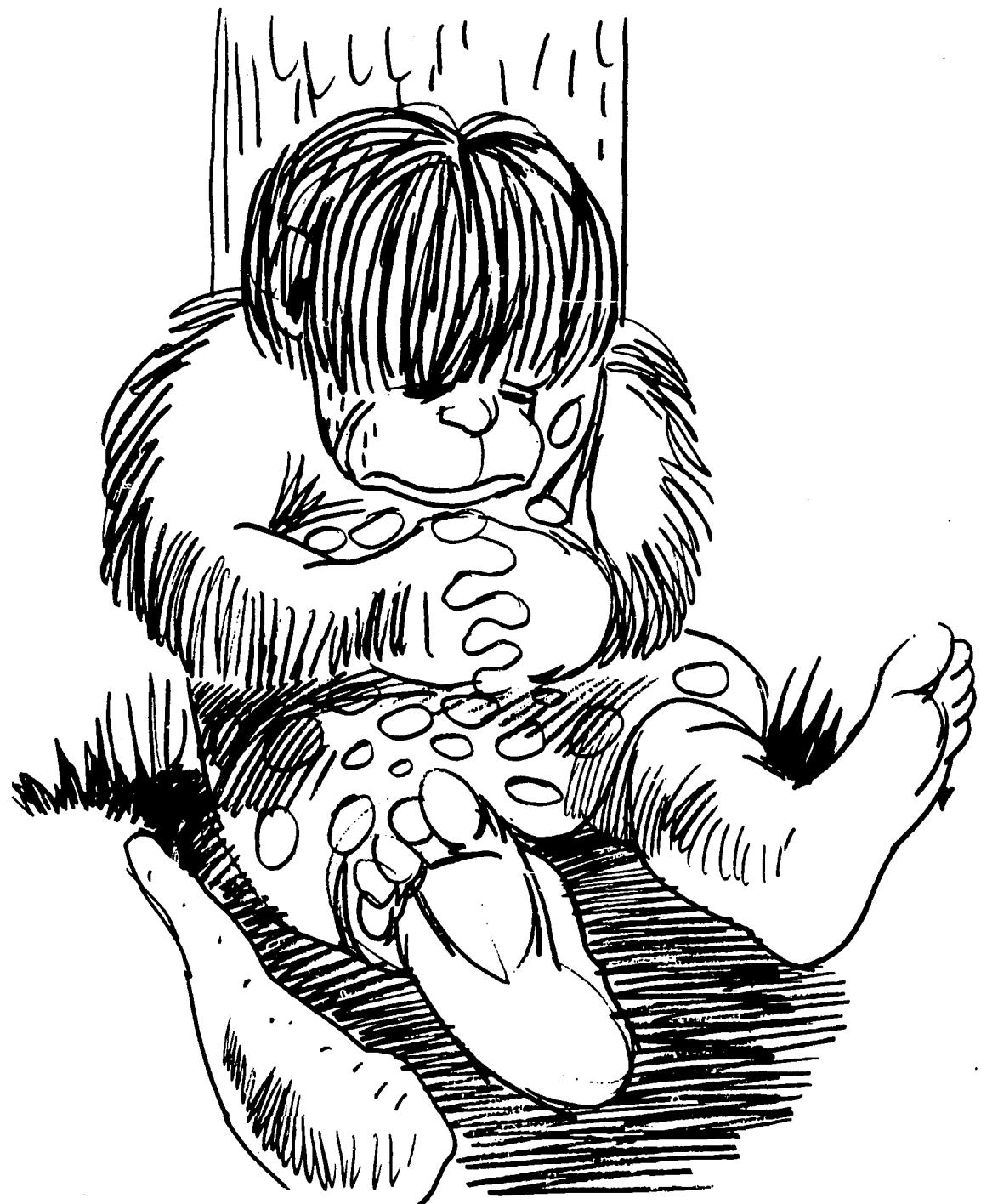
"What do you want?" growled the chief.

"I want to give you this bag of stones," said Ugboo. "There is one stone in this bag for every sheep in the flock and one sheep for each stone. There are just as many stones as sheep. You can see for yourself."

The chief went back with Ugboo and began matching the stones with the sheep. Sure enough, there was one stone for each sheep and one sheep for each stone.

Zarathustra muttered to himself. "Let's go over this again very slowly. I want to be sure, really, really sure I understand this. Let me see. One stone for each sheep. Then there are at least as many stones as sheep."

## UGBOO'S BIG PROBLEM



"This thinking business is hard work. . . I had better rest awhile."

## UGBOO'S BIG PROBLEM

He sat down and wiped the sweat from his brow. He said to Ugboo, "This thinking business is hard work. I'm tired already - I had better rest awhile."

After a few minutes he said, "Where was I? Oh, yes. There are at least as many stones as sheep. But there might be more stones than sheep. We might have one stone for each sheep, and still have some stones left over. Let us look. Are there any stones left over?"

Zarathustra started to look to see whether there were any stones left over. Ugboo reminded him, "Don't you remember, chief? There is one sheep for each stone. So there can't be any stones left over."

Zarathustra was puzzled. He wrinkled his brow and thought. He thought very, very hard. Suddenly he began to smile.

"That's right!" he said. "That's absolutely, positively right! It is hard to think of all these things at one time. But once you do put it all together, it is really easy to see. Sure, one stone for each sheep. So there are at least as many stones as sheep. One sheep for each stone. So there are at least as many sheep as stones. As many stones as sheep. . . as many sheep as stones. . . ." He looked from the stones to the sheep, to the stones, to the sheep, and so on for another few minutes.

UGBOO'S BIG PROBLEM



"Of course! There are just as many stones as sheep."

## UGBOO'S BIG PROBLEM

Suddenly he pounded one fist into the palm of his other hand and shouted, "Of course! There are just as many stones as sheep!"

Then he turned to the boy and asked, "So what? Why are you taking up my whole morning with all this stuff? Why haven't you taken the sheep out to the grassy place yet? They must be hungry already."

Ugboo answered, "Don't you see? When I bring the sheep home tonight, you can match the stones with the sheep again. You can put down one stone for each of the sheep. If there are no stones left over, then you will know that I have brought back all the sheep. If there are some stones left over, you will know that I have lost some sheep, and you can punish me."

Zarathustra said, "How is that again? If there are some stones left over, then. . . . Let me think it over. . . . Oh, don't bother me any more! Go tend your sheep right now, or I won't wait until tonight to punish you. Get going. Scoot! I have to go hunting. Do you think I have all day to talk to little boys?"

## UGBOO'S BIG PROBLEM



During the hunt, Zarathustra told the other men about what had happened.

## UGBOO'S BIG PROBLEM

During the hunt, Zarathustra told the other men about what happened.

"You know that boy, Ugboo? He came to me this morning with a long story about stones and sheep."

The chief began explaining to the men about one stone for each sheep and one sheep for each stone and, sure enough, he got it all mixed up.

"I'm sure I understood it this morning," he said. "Now I am so dizzy, I am not sure of anything. Let's hunt now and talk about it at supper tonight."

UGBOO'S BIG PROBLEM



The men of the tribe sat around the fire eating legs of lamb.

## UGBOO'S BIG PROBLEM

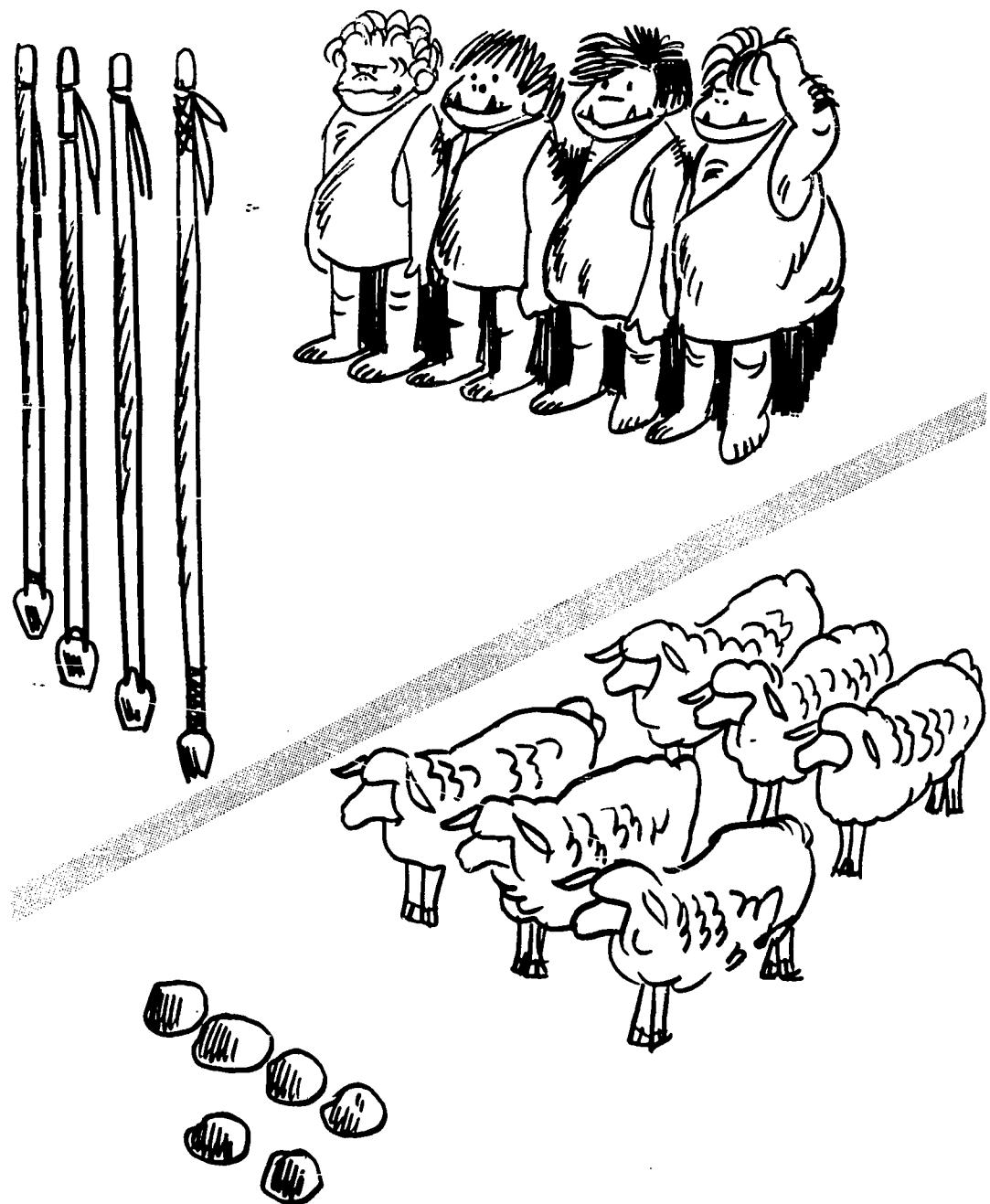
That night the chief talked it over with all the other leaders in the tribe. They were sitting around the fire in a circle, each one biting into the leg of lamb he held in his hand, and they were arguing at the top of their voices. "Wait," one shouted. "Suppose you had some sheep left over, then . . . ." Another yelled back, "Don't you yell at me! What if . . . ."

Finally, one of the men said, "Let's call Ugboo over, and have him tell it to all of us. I'm sick and tired of all this shouting and arguing."

So Ugboo was called. His mother and his brothers and sisters crowded around the edge of the circle and watched and listened. A little boy like Ugboo was never asked to join the circle of men. His mother was worried and thought, "Has Ugboo done something wrong? Will the chief punish him? Oh, what will happen to him now?"

Ugboo very slowly explained about the stones and the sheep to all the men. After he finished they were quiet for a while. Then they began asking questions.

## UGBOO'S BIG PROBLEM



Just as many men as spears. Just as many sheep as stones.

## UGBOO'S BIG PROBLEM

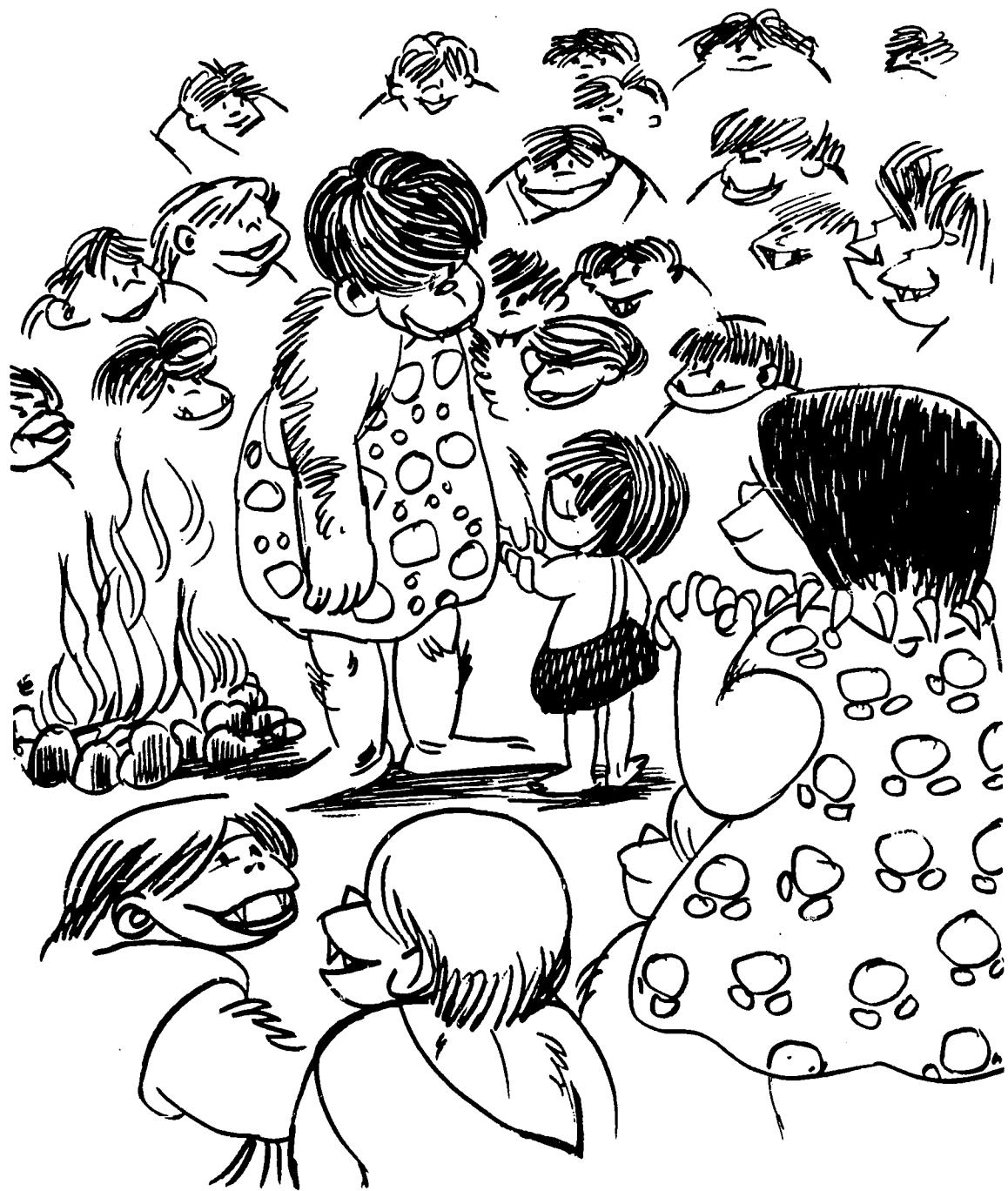
"Would this work with other things, too?" one of them asked. "If there is one spear for each man, and one man for each spear, then are there just as many spears as men?"

Another man asked about people and noses. Another one asked about knives and bones.

At last Zarathustra said, "This is a wonderful discovery. If we have two sets and want to know which one has more in it, all we have to do is pair one thing in the first set with one thing in the other. We keep on matching one for one. When we are done, if there are any left in one set, then we know that that set has more. If there is nothing left over in either set, then there are just as many in one set as in the other."

All the men agreed that this was a very good idea. One of the men said, "Ugboo is a very clever boy. We ought to do something for him." The others said, "Yes, he deserves some reward."

## UGBOO'S BIG PROBLEM



Chief Zarathustra was very pleased with Ugboo.

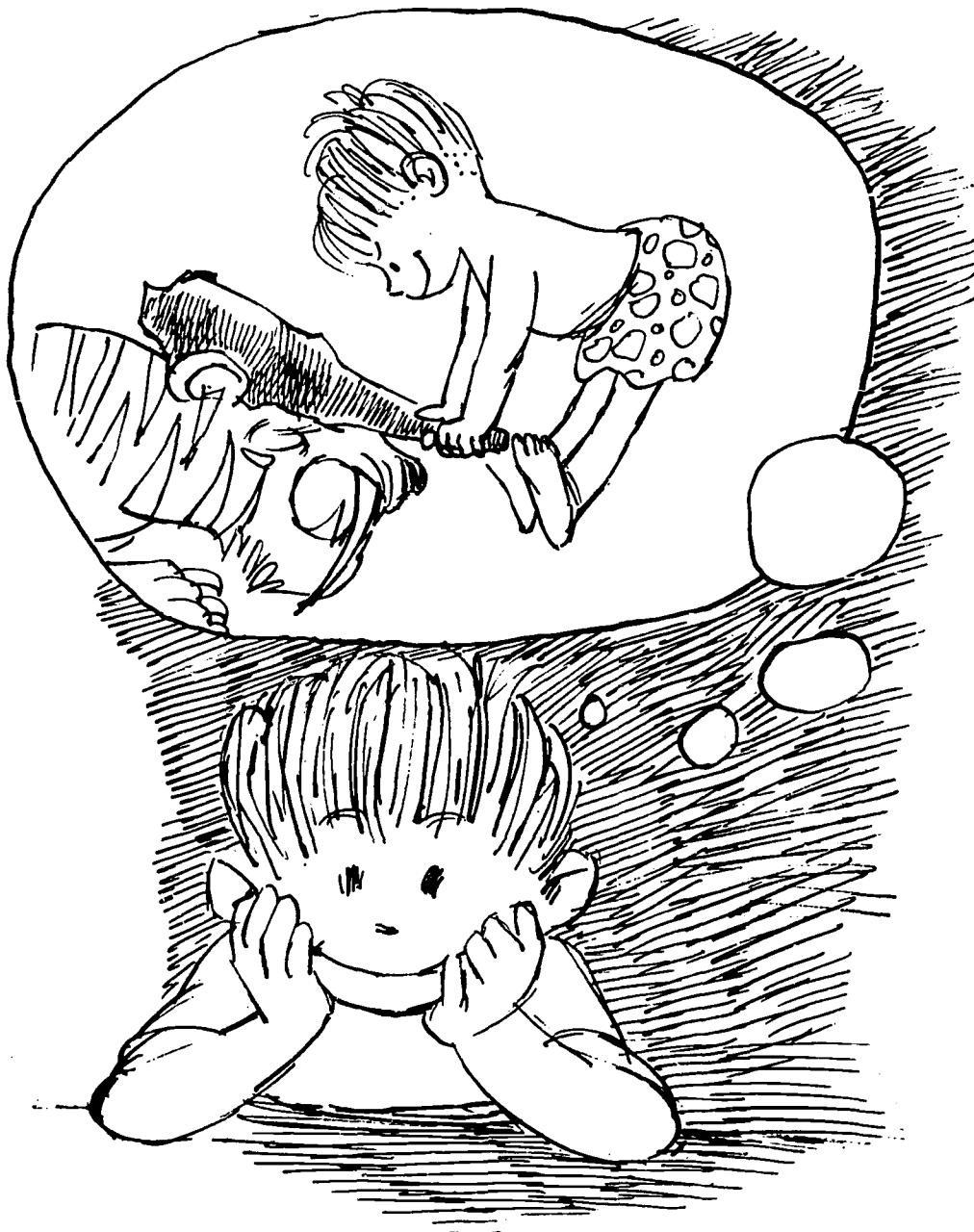
## UGBOO'S BIG PROBLEM

Then the big chief, Zarathustra, said to the boy, "We are going to give you a great honor. We will give you a long name like all the big men have, so that every one will know that you are an important person. From now on, we will call you Uggabugboo."

Uggabugboo's mother and brothers and sisters all ran up to him and hugged and kissed him. His father patted him on the back and shook hands with him. No other boy in the whole tribe had ever had such an honor.

When he grew up, Uggabugboo became a very big chief himself. Then he was called Uggabuggabuggabugboo. He was really important then.

UGBOO'S BIG PROBLEM



David liked his father's story about Ugboo and his discovery.

## UGBOO'S BIG PROBLEM

When David's father had finished reading the story, David was quiet for a while.

Then he said, "Thanks, Dad. That was an interesting story, but I don't see why they made such a fuss. Now that I know about matching one-for-one I see that it is a very simple idea. What I can't understand though, is why I never thought of it before."

His father smiled wisely and said, "Some of the greatest ideas are simple, once somebody thinks of them. What Ugboo discovered is one of the most important discoveries ever made. Keep on thinking for yourself, David. Maybe someday you will discover something nobody else has ever thought of.

Historical basis for the theme of Ugboo's Big Problem is to be found in a book authored by Richard Starr, Harvard University Press, 1939, and entitled Nuzi: Report on the Excavations at Yorgan Tepa, near Kirkik, Iraq.

The report notes the finding of "a large number of inscribed clay texts, one of which was the extraordinary tablet of 'Zikarri the shepherd'." Roughly egg-shaped and hollow, with a hole at its pointed end, it bears the inscription "stones of the sheep" followed by a list of animals given to "Zikarri the shepherd", presumably for grazing. Inside the tablet were forty-nine pebbles.

## Suggested Activities on Constructing and Describing Equivalent Sets

1. Have a child pretend to be "Ugboo" and put a stone in a bag or box as each sheep (child) leaves the fold. Then have another child, as "Zarathustra", take a stone out of the bag as the sheep return to the fold. The activity could be varied by having one or more sheep hide, pretending they are lost. When it is discovered that there are stones for which no sheep appear, the teacher might ask such questions as, "Are there more stones than sheep? Are there fewer stones than sheep?" After this has been determined the child who is Ugboo should search for the lost sheep.

★ 2. Tell the children to pretend that some flannel board cut-outs are Ugboo's sheep and some beads are stones. The flannel board is the pen. Put some sheep in the pen in full sight of all the children. Choose a child to be Ugboo. He picks up as many stones as there are sheep in the pen and gives them to another child, "Chief Zarathustra". The chief then removes one sheep from the flannel board and places it on a table. He puts down one of the stones beside it. He continues this procedure until he establishes that there are as many stones as sheep and vice versa.

After the set of sheep has been matched, rearrange the sheep and ask the child, "Now how many stones should there be, more, less, or the same number as before?"

This activity should be repeated with other sets until it is obvious to the children that rearranging the members of a set does not require changing the set of stones used to represent the number of the set.

★ See footnote on page v.

Next substitute another cut-out of a lamb for one of the larger sheep. Ask the class if this set of sheep is the same as the one before. (It is not the same set.) Next, ask the children if they now need more, less, or the same number of stones to represent the number of this new set of sheep. This may be repeated using other sets of objects in order to bring out the fact that the number of a set (as represented by another set) is unchanged if one member is replaced by another.

- ★ 3. Ask the children to think of other ways of describing "how many" without counting or using number words. Show the children a set and ask them how they might describe "how many". They might suggest constructing a matching set of fingers, tally marks, or sounds (there are bop-bop-bop-bop objects). Ask them to describe a familiar set having the same number of members. Example: "There are as many objects as there are wheels on a tricycle".
- 4. Select 5 children (by name) to stand in one part of the room. Call them set A (or any other name). Have 3 other children stand in another part. Call them set B.

Ask the class to tell which set has more members.

Ask how they know. Example: Mary can be matched with Jill, Bill with Alice, Jim with Steven, but there are no members left in B to match with Peter and Susie, so A has more members.

Choose someone to put more members into B to make a new set C having the same number as A. Although these sets have the same number of members, they are not the same set because the members in A are entirely different people from those in C.

Correct statements are:

"A has as many members as C"

or

"Both sets have the same number of members"

or

"The sets can be matched"

or

"A and C are matching sets"

or

"The sets are equivalent"

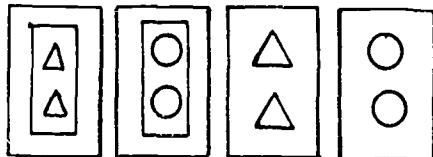
## ★ 5. Game "Info"

Select 8 different Color Form cards from the deck. This selection of cards should be made such that:

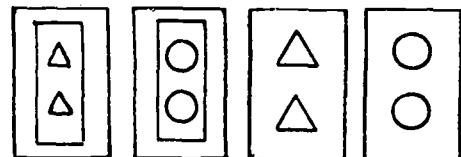
- 1) half of the cards are of one color and half of another color.
- 2) half of the cards are unframed and half are framed.
- 3) half of the cards have one design while the other half has another design.

For example, the following cards might be used:

RED



GREEN



Place the cards on the chalk tray in front of the room and select one child to be "Mr. Info". Mr. Info is given 3 counters and a small piece of chalk. While the rest of the class covers their eyes, Mr. Info hides the piece of chalk behind one of the cards. After hiding the chalk, Mr. Info then calls on someone in the class to be a guesser. When the guesser stands up to guess which card the chalk is behind, Mr. Info must give him the three counters.

Each time the guesser asks a question, he must return one of the counters to Mr. Info. Therefore, when the guesser has returned all of his counters he may not ask any more questions, and Mr. Info must select a new guesser. When a guesser discovers the card with the chalk behind it, he becomes the new Mr. Info.

After several games, discuss the type of questions that are good to ask.

**Note:**

The strategy of this game is to narrow the set of possibilities down by asking questions which will partition it into equivalent subsets. for example, "Is it a red card?" "Does it have a frame?" or "Does it have a circle on it?" Do not reveal this strategy to the children. If the child discovers it and explains it to the class, this is acceptable, but do not push the children to verbalize. However, this game is repeated in later units and it is an important discovery for the children to make by themselves.

If the children learn the strategy for 8 cards, (as evidenced by their using only three appropriate questions, do not ask for verbalization) add two more colors to make 16 cards, and allow each child four questions.

If the children learn the strategy for 16 cards, substitute children for cards. That is, have 16 children stand in the front of the room, with Mr. Info whispering the name of one of them to a referee before calling upon a guesser.

Five guesses are sufficient for up to 32 children.

## Commentary on Worksheet 1

The sets of fruit are different, and it will be interesting to see if any child in the class discovers this difference. Direct the children to look at the two pictures.

Ask:

"Who can name one fruit which you see in the pictures?"

"Is this fruit in both pictures?"

"Who can name another one of the fruits?"

Continue questioning until the names of all the fruits have been mentioned. It is hoped that some child will notice that there are more bananas in one picture than in the other.

Ask:

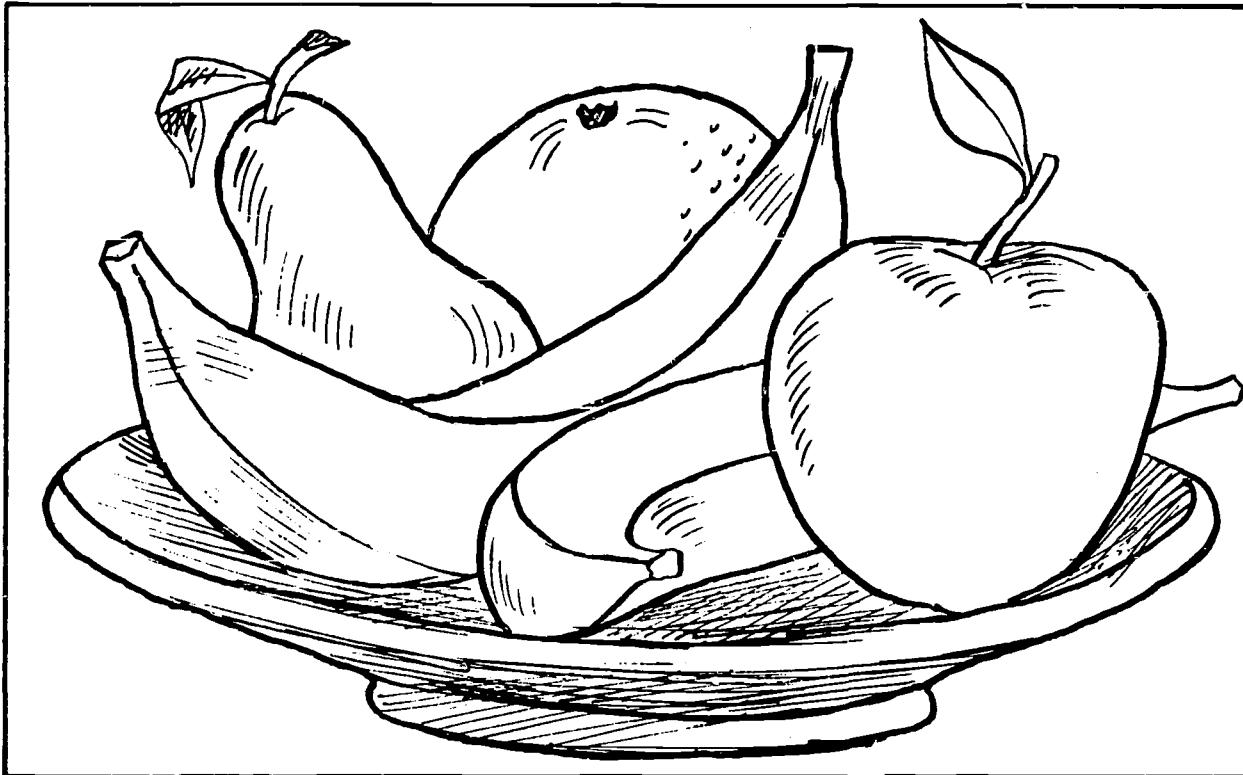
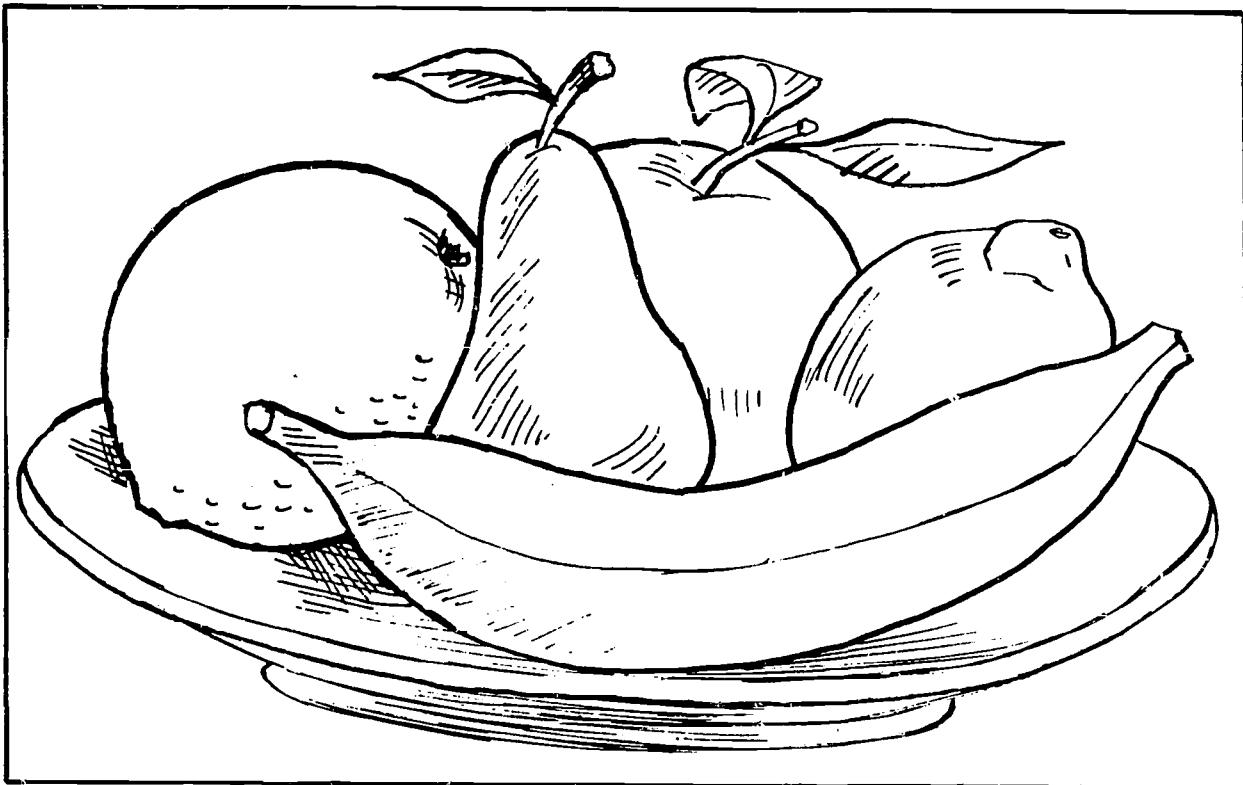
"Are the two sets the same?" (No)

"Why?" (One has more bananas than the other.)

"Do the two sets have the same number of members?" (Yes)

"How do you know?" (They can be matched.)

★ See footnote on bottom of page v.

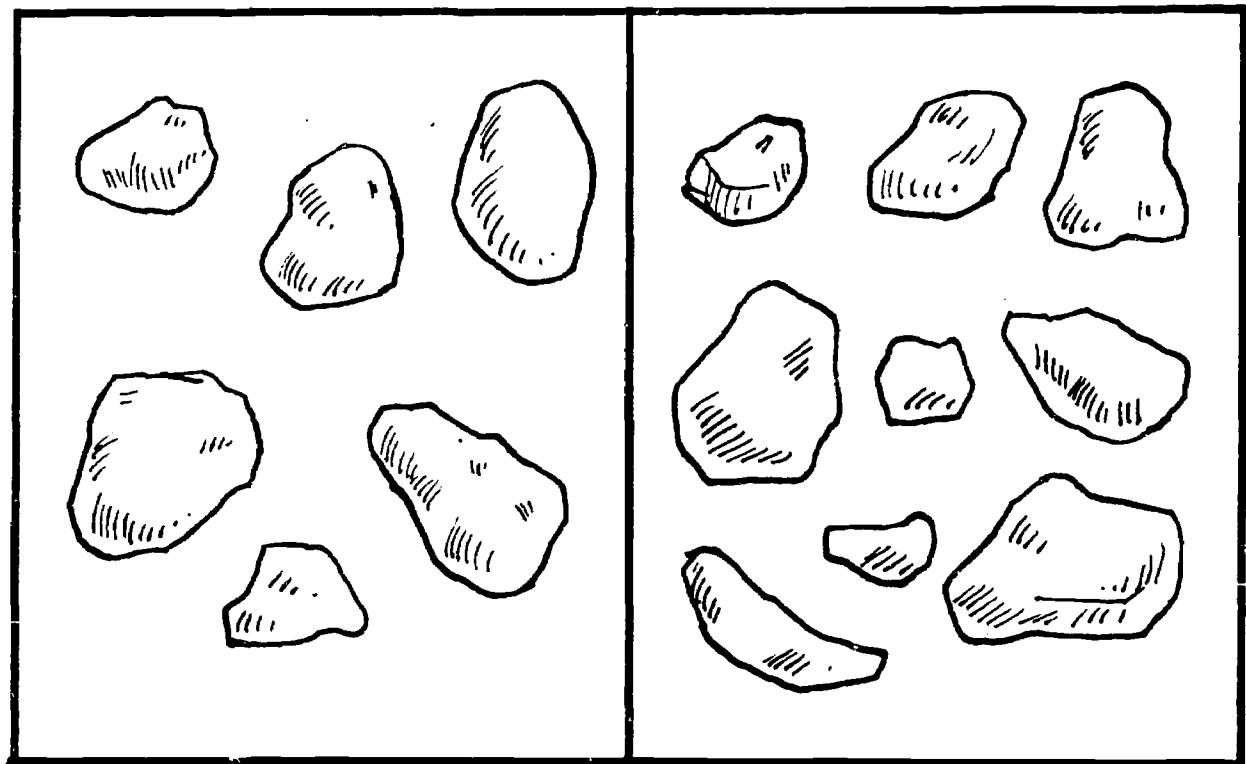
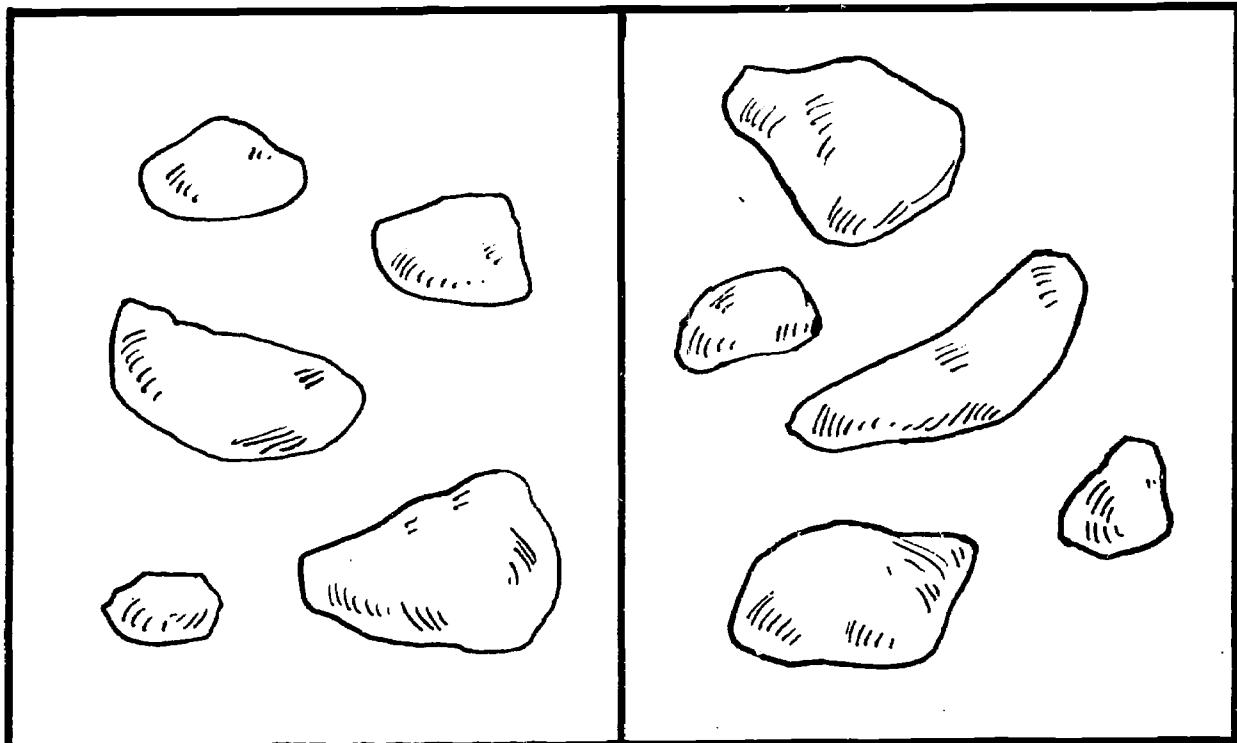


## Commentary on Worksheet 2

This is another exercise in estimating. Have the children look at the two pictures in the top row. Then ask the following questions:

1. How many think the top two sets have the same number of stones in them?
2. How many think one set has more stones in it than the other? Which do you think has more?
3. Put your finger on the set that you think has more.
4. How could we find out if these sets have the same number of stones in them? If some child says, "Count them", your reply could be, "Yes, but let's pretend we don't know how to count. Then how could we check?" (By drawing connecting lines.) Using different colors may make it less confusing.
5. How many have connected all the stones?
6. What does this tell us? (That both sets have the same number of stones.)

Follow this procedure for the bottom sets.



## Commentary on Worksheets 3, 4, 5

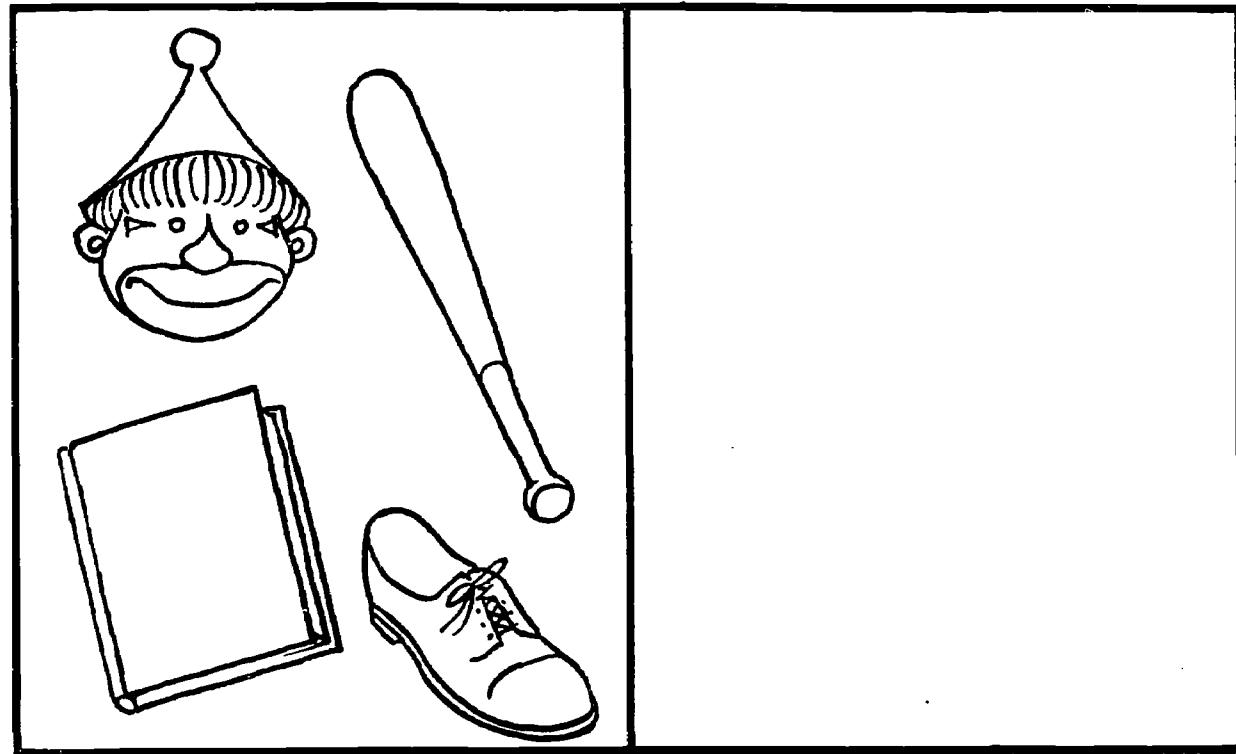
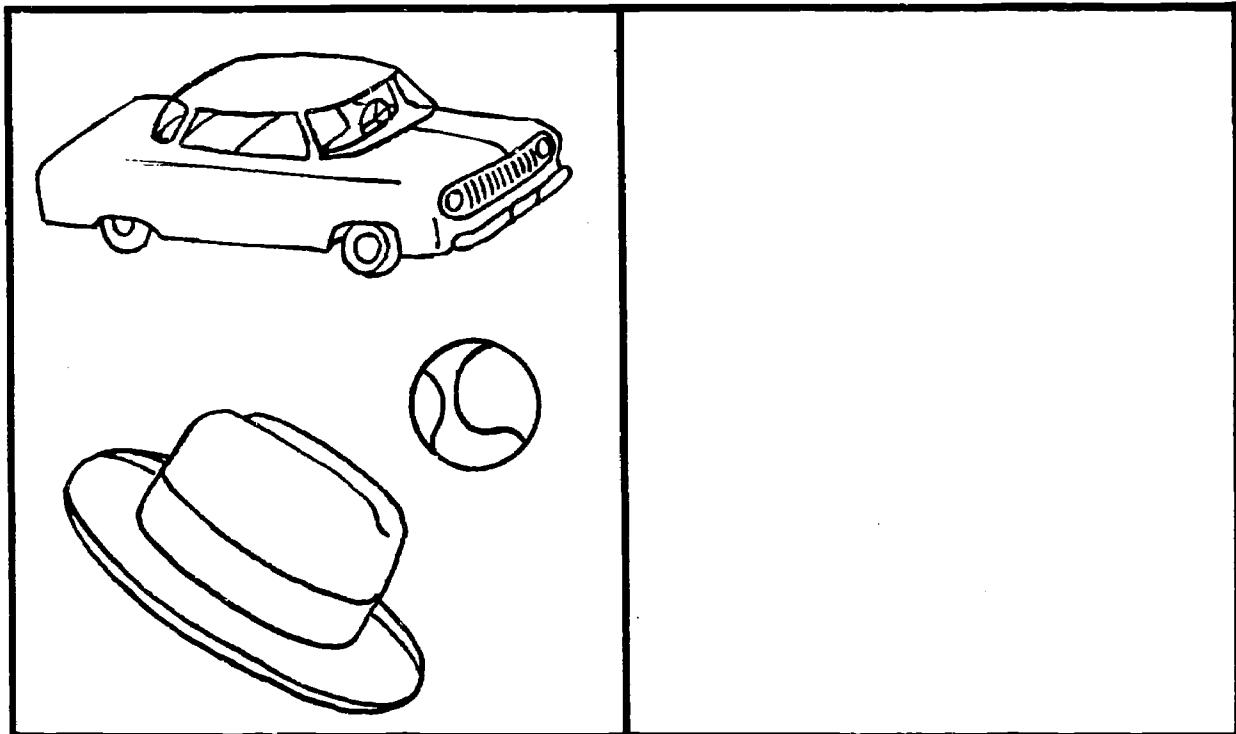
These worksheets are designed to give the children practice in constructing and matching sets, and tallying. They also review open and closed curves.

Distribute Worksheet 3. Instruct the children to draw a set in the empty box on the right to match the set in the box on the left. It is necessary only to have the same number of members in each set; the members do not have to be identical. If, however, a student draws an identical set, this is acceptable since he would then have the appropriate number of members in his set.

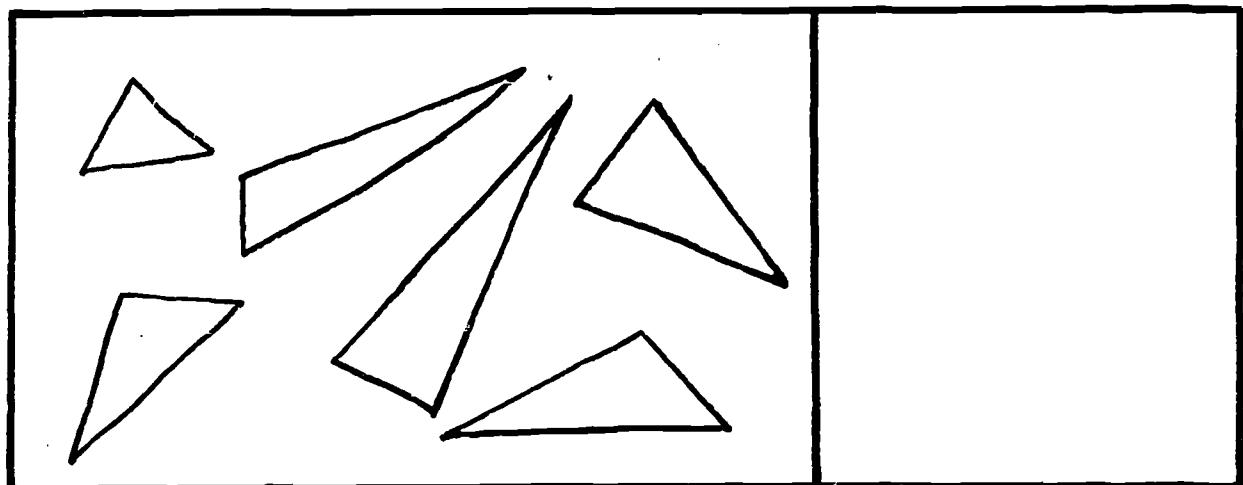
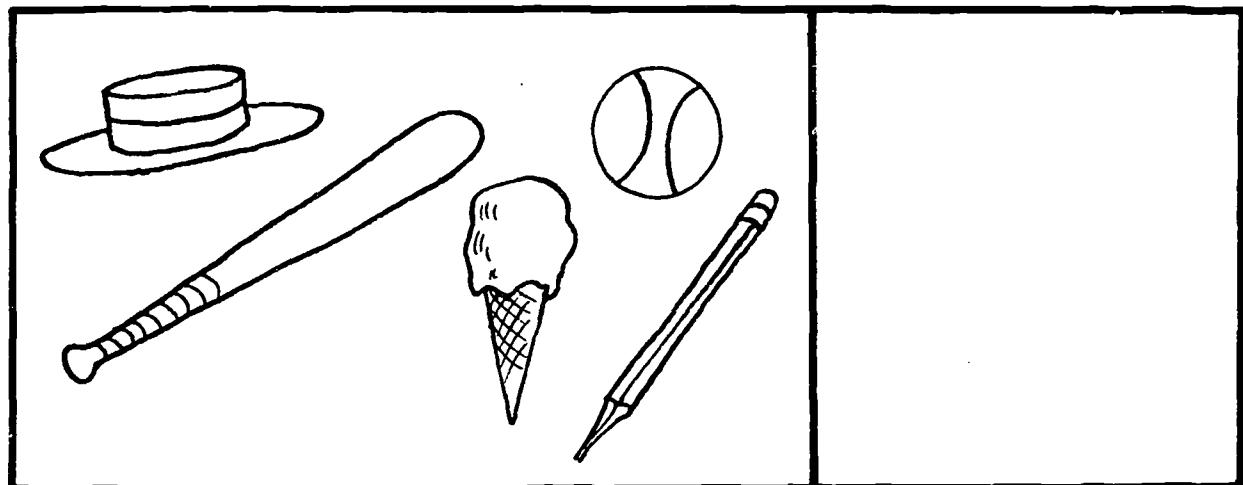
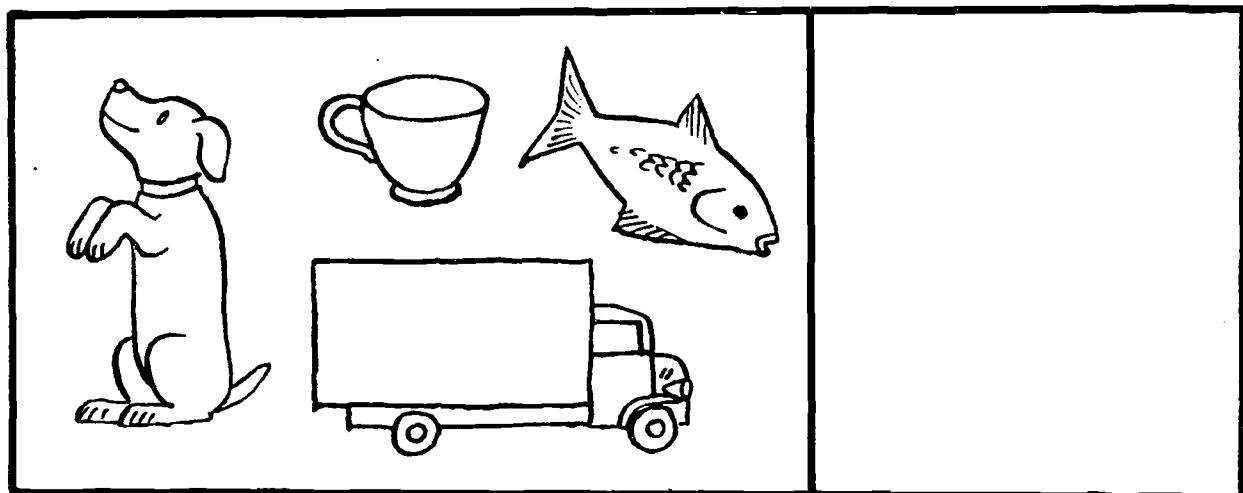
The children can check their own answers by drawing lines from each member in the original set to a member in the new set. If there is a one-to-one correspondence, the sets have the same number.

Worksheet 4 deals with tallying. The children are to make a tally mark in the empty box on the right for each member of the set in the box on the left. Before distributing this worksheet, demonstrate on the board how to tally the number of members in a set. The tallying of groups by five is not introduced at this time, but is acceptable if the children do it. A set consisting of six members might be tallied:    |||    |||

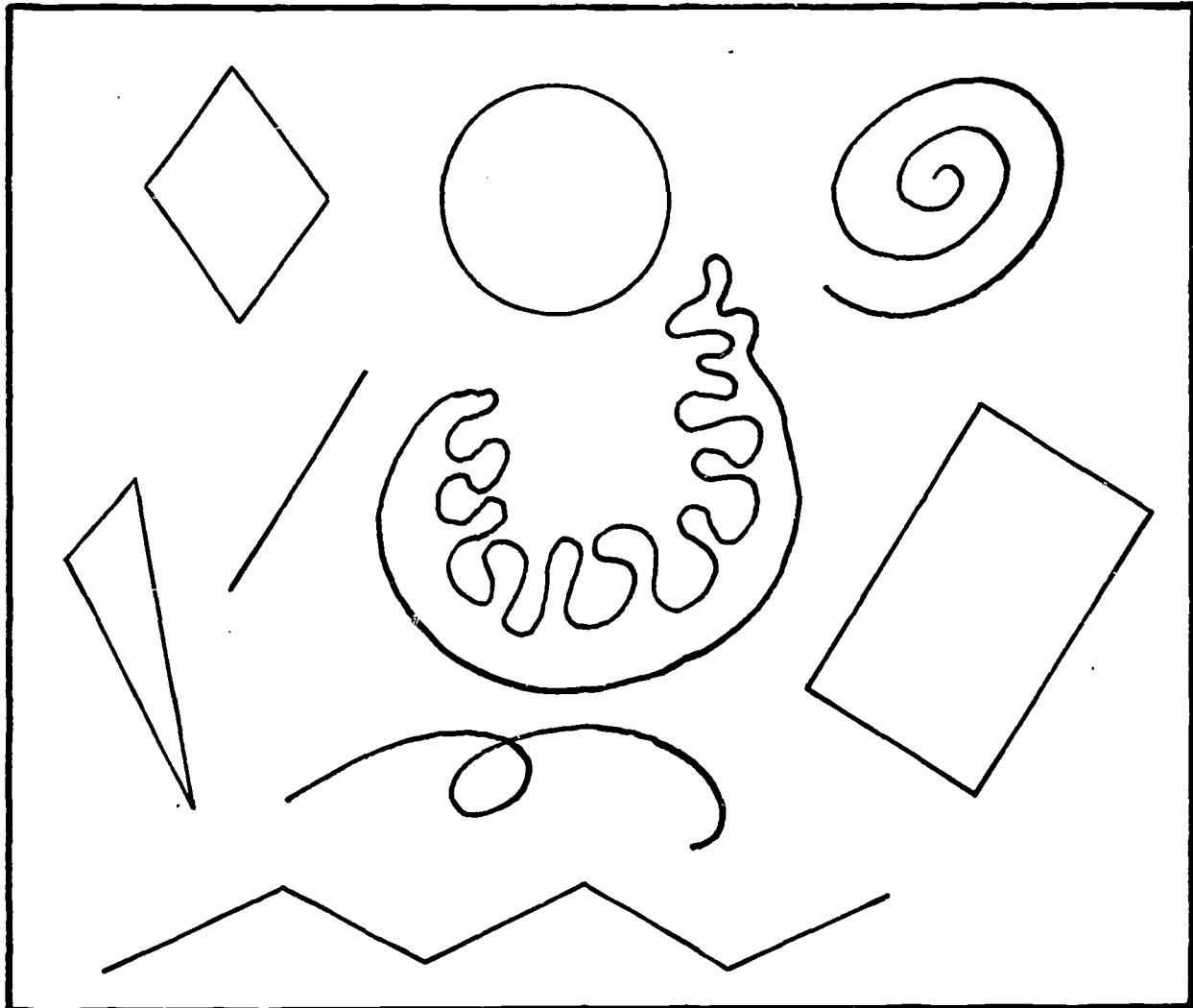
★ Worksheet 5 is another exercise in tallying and reviews open and closed curves. It may be necessary to review open and closed curves prior to the distribution of the worksheet. Read the directions on the worksheet, allowing for completion of the first assignment before reading the next instructions.



Draw a set in the empty box to match the set in the box on the left.



Make a tally mark in the empty box for each member of the set on the left.



Make a red tally mark for each simple closed curve.

Make a green tally mark for each simple open curve.

Make a brown tally mark for each simple curve.

### Commentary on Worksheet 6

Hold up a copy of Worksheet 6 for the class to see. Then put it down. Say to the children, "Take out as many counters as you think you will need if you are going to put one on each man shown on this paper." Have them put the rest of their counters away.

Pass out the worksheet. Have the children put one of the retained counters on each man. Check the correctness of their estimations by posing the following questions:

"How many of you have counters left over?"

"How many did not have enough to cover every man?"

"How many had just the right number?"

Then ask, "Are there more spears than men, more men than spears, or the same number?"



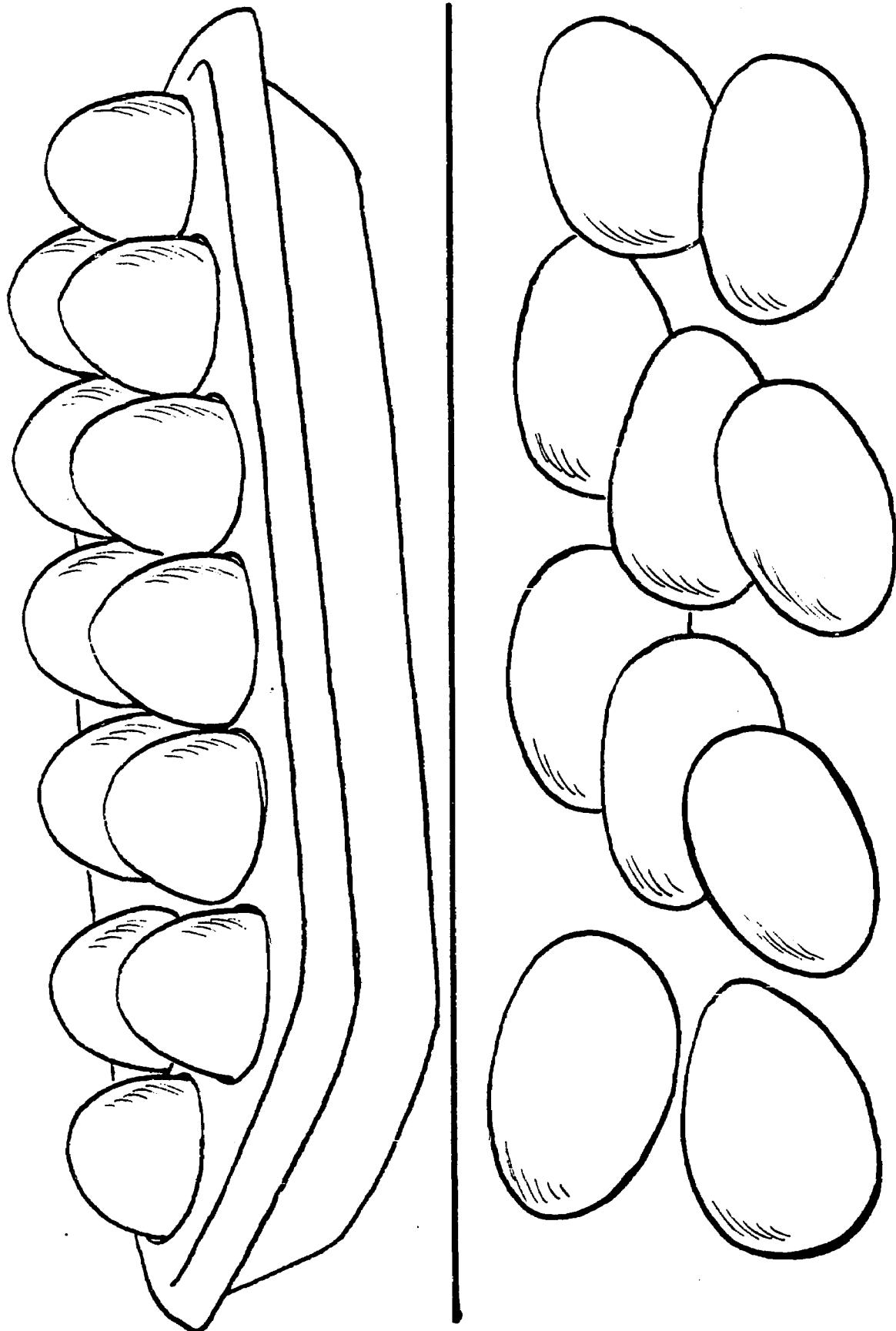
### Commentary on Worksheet 7

Have the children raise their hands if they think the box has more eggs in it than the loose set of eggs on the right. Next, ask for a show of hands from those who think there are more eggs in the scattered set than in the box. Then ask how many think that both sets of eggs contain the same number.

Ask the children to cover each egg pictured on the right with a checker or other marker. Then have them transfer these markers, one at a time, to the eggs in the box.

Ask: "Are all of the eggs in the box covered with a marker?"

Have them determine which set has more eggs - the sets of eggs in the box or the other set (the box).



## Commentary on Worksheet 8

This worksheet reviews linear measurement and introduces the terms "tallest" and "shortest". The two sets of children are equal sets since they are the same children.

Illustrate this in the class by choosing children of four different heights. Pretend to take a picture of them. Then have the children arrange themselves in a different order. Pretend to take another picture. Ask the class the following questions:

"Is this the same set of children?" (Yes)

"Are there as many in the second picture as in the first?" (Yes)

"Is the same child the tallest one in both pictures?" (Yes)

"Is the same child the shortest one in both pictures?" (Yes)

### Top Picture

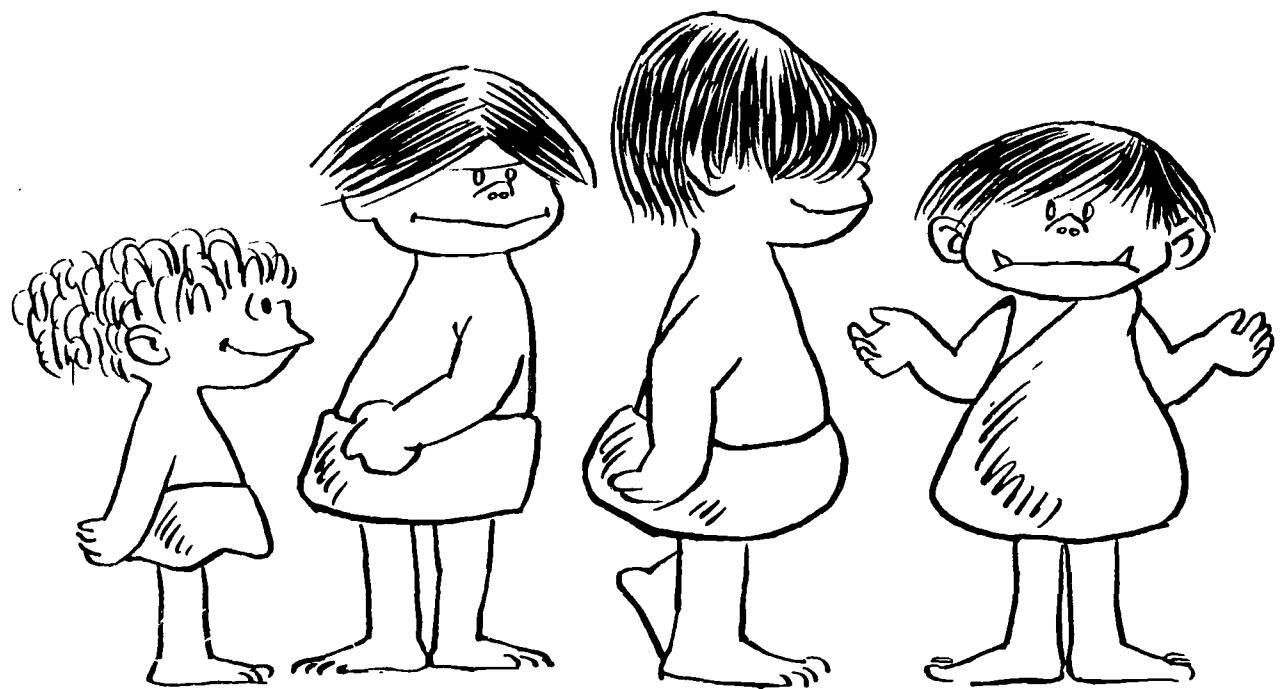
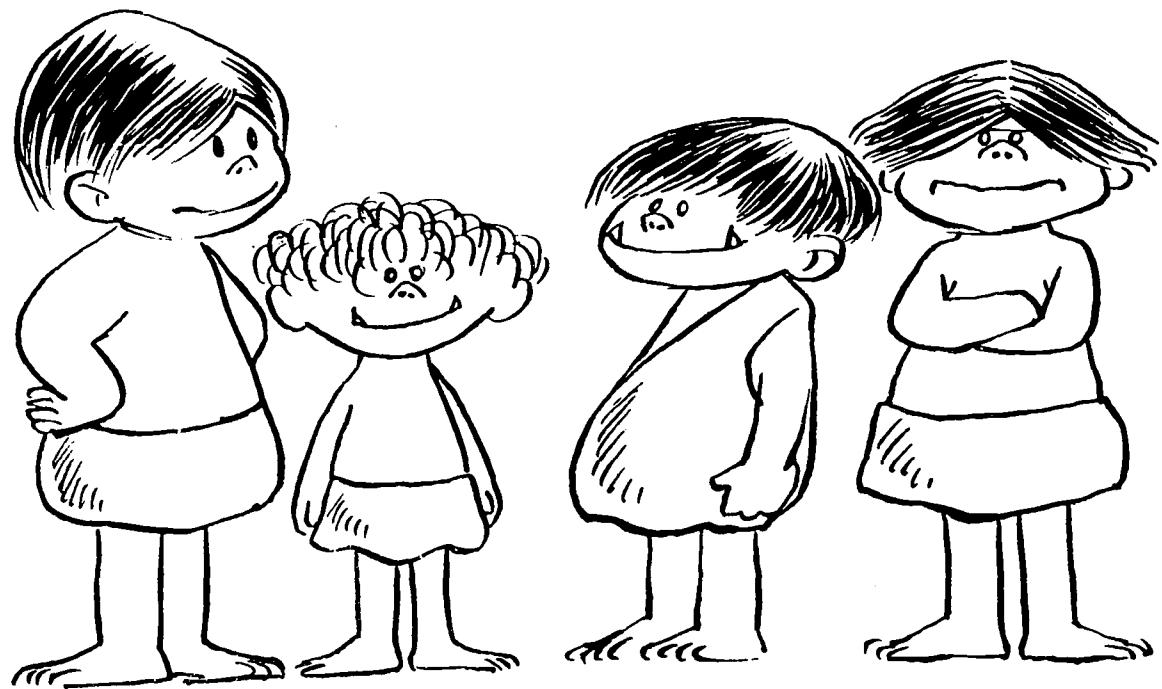
Tell the children to:

1. Look for the tallest child and color his clothes red;
2. Choose the shortest child and color his clothes green.

### Bottom Picture

Tell the children to:

1. Look for the tallest child and color his clothes red;
2. Choose the shortest child and color his clothes green.



In both pictures:

1. Color the tallest child's clothes red.
2. Color the shortest child's clothes green.

## Teacher Background on Minnebars

The following activities require the use of several boxes of "Minnebars". Minnebars are small wooden bars which are cut according to unit lengths. When we speak of "units of length" we mean that an arbitrary length has been chosen as our standard unit and that each Minnebar in the box is made up of one or more of these units. A set of Minnebars should contain at least 10 one-unit bars, 5 two-unit bars, 4 three-unit bars, 3 four-unit bars, and two each of the lengths five through ten units. In other words, ten different lengths of bars may be found in each box. The bars of each different length also have different colored surfaces. The bars are not all colored, providing a means of reducing the child's dependence on the colors of the bars and forcing him to concentrate instead on the lengths of the bars. For the beginner, however, the color identification method of playing the games can be helpful.

With the use of these bars the teacher can expose the children to countless mathematical procedures and concepts. Although the children may not verbalize their processes of thinking in strict mathematical terms such as "add", "subtract", or "equal" in playing these games, they are, nevertheless, employing rather mature processes of mathematical logic as they play. The Minnebars come to be the young child's symbols in solving mathematical problems. The mathematical labels for the processes can easily be added at a later point in the child's development.

Among other things, the following games, in conjunction with the Minnebars, may serve to expose the children to the thought processes used in addition, subtraction, doubling, numeration, and proof.

The teacher is not expected to teach these mathematical concepts directly. Rather, she will find that these concepts are almost self-taught through discovery as the children adhere to the rules of the games.

It is not intended that the blocks be pulled out only at this time and then put away for the rest of the year. Rather, the children should continue to come back to them throughout the year, playing new games as they are interested.

## Activities Using Minnebars

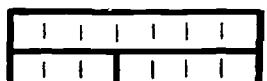
1. Permit each child to examine and play with the Minnebars, thus allowing them to become familiar with the bars.

★2. Place two bars (each no larger than the five-unit bar) end to end, and ask the children if they can find just one bar that will "match" these two bars in length. (In other words, the correct bar would be just as long as the two bars put together.) The teacher may need to demonstrate this.



The verbalization of the number of units per bar should be minimized at this point, and, instead, the colors of the bars should be used as a means of identification where necessary. (For example, refer to "this bar" or the "yellow bar" rather than the "eight-unit bar".) Also, the children need not always have the unit division marks on the bars visible.

★3. Place two bars, one shorter than the other, side by side  and ask the children to find the bar which will complete the match.

★4. Ask the children to match one long bar with several shorter bars. Any combination or number of shorter bars may be used to match this one large bar. For example, a seven-unit bar may be matched in this way:  or, perhaps like this: 

Ask the children to see how many different ways they can match a long bar. With the nine-unit bar, for example, there are enough ways for each child in the class to match the bar in a different way.

★5. Show the children a six-unit bar that is matched with 2 three-unit bars. Point out that this bar can be matched using bars of only one color. Ask if they can find another way to match the six-unit bar using bars of only one color. (They could use two-unit bars or one-unit bars.)

Lead the children to discover that all the bars can be matched using only the one-unit bars.

Next, have the children investigate which of the larger bars can be matched using smaller bars of only one color other than the one-unit bars.

6. Have the children discover which bars can be matched using only the two-unit bars, and which bars cannot be matched in this way.

★7. Next, ask the children to match a ten-unit bar, for example, using just 2 bars. (This particular bar could be matched using a three-unit bar and a seven-unit bar.)

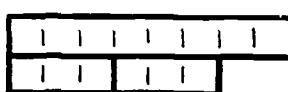
Following this activity, ask the children which bars can be matched using just 2 shorter bars. (All of the bars except the one-unit bar can be matched in this way.)

★8. Have the children discover which bars can be matched using two bars of the same color, and which bars cannot be matched in this way.

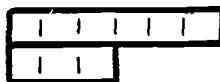
9. Activity 2 might be played using 4 bars (including the large bar) in the match, rather than three. In other words, place three short bars end to end and ask the child to find just one bar to match your string of bars.

10. Activity 3 might also be played using 4 bars. In this case, supply the large bar and two of the three bars to be used in matching it. Then ask the children to find one other bar to complete the match.

For example, supply:



11. Next, supply only one of the three shorter bars and ask the children to find 2 other bars to complete the match. For example, supply:



12. Finally, supply just one large bar and ask the children to find 3 shorter bars that will match your larger one. To match this bar the children will place their shorter bars end to end.

13. Children can play this game in pairs or teams. The players begin with a long row (not more than twenty; start with short rows) of one-unit bars placed end to end and take turns removing bars until they are all gone. Each time a player either:

- a) removes one bar, or
- b) finds two longer bars that match each other and that will match the original row when placed together, and then removes all the one unit bars that match one of the new longer bars. This essentially results in halving the original row. However, an explanation of the game in terms of matching may be more useful at this point.

The player who removes the last bar wins.

## Introduction to Mathematical Golf

**Note:**

Earlier in the Unit we had the children construct or describe sets to serve as numerals. This is a valuable experience since by studying, or better still constructing, a different numeration system, children gain insight into the structure and operation of the normal Hindu-Arabic system.

Although the following activity is basically a matching game, the rules of the game develop another numeration system. This new system satisfies an essential criteria for all numeration systems: no two different numbers may be described by the same symbol.

The game is a coding system in which two children work together to discover the appropriate symbol in this new numeration system for each of 10 different lengths of bar. To make the game more exciting, each different length of bar is called a "target" and each rule used in matching a bar is called a "shot"; the teacher may use different terminology if she wishes. The children attempt to find the correct series of shots which will "hit" (or match) the target.

The game should be approached simply as a game with the bars. The teacher should make no mention of number or numeration.

The children play this game in pairs with one child designated as the "shooter", the other as the "scorer". Each pair plays as follows:

## The Shooter

The Shooter is given a large Minnebar 4 to 10 units in length. This bar is called the "target". The Shooter is asked to "hit" the target by matching it in length with several smaller bars.

The Shooter must always begin each game with a large bar placed in front of him as a target, and a one-unit bar placed in front of the target in the "shooting row":

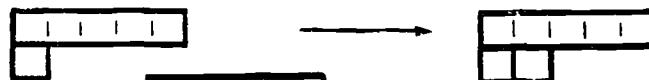
Target        
Shooting row

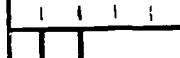
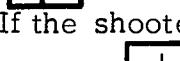
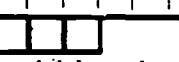
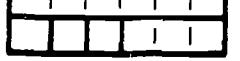
At each move the shooter must choose between two kinds of shots to use:

1) The "Match Shot":

The Shooter finds one bar that will match his whole shooting row, and places that bar at the end of the others in the shooting row.

This is the shot the children must use first (let them discover this necessity). They find a one-unit bar to match their starting bar and place it at the end of their shooting row:



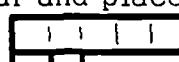
If the Shooter had  , the match shot would produce  . If the shooter were trying to match the six-unit bar, and had progressed to  , the match shot would produce  , which would hit the target.

In order to score a hit, the target must be matched exactly. The shooting row may never extend beyond the target.

The four- and eight-unit bars can be hit using only this kind of shot.

The teacher may wish to have the children try hitting these two targets before she teaches them the other kind of shot.

2) The "single shot":

The shooter finds a one-unit bar and places it at the end of the shooting row. If the shooter had  , using the single shot would produce  .

The single shot can be used only under certain conditions determined by the "scoring", while the match shot can always be used, even when this kind of shot makes it impossible to hit the target.

The Scorer

The Scorer (or record keeper) plays with only two lengths of bars: one-unit and two-unit bars. He begins each match with a two-unit bar placed

vertically in front of him  . The scorer has two important tasks:

- 1) Whenever the Shooter matches his shooting row, the scorer places a one-unit bar at the end of his "Scorer's Row" like this:  . If the Shooter then matches his shooting row again, the Scorer's row becomes  .
- 2) Whenever the Shooter wants to make a single shot by placing a one-unit bar in the shooting row, the Scorer must look at the last bar in his own row. If it is a one-unit bar, the Scorer removes it and replaces it with a two-unit bar. If, on the other hand, the Scorer has a two- rather than a one-unit bar in the last position in his row, he tells the Shooter that he cannot make that kind of shot; the Shooter can then use only the match shot (or start over).

Suppose the Shooter has the following bars in his rows:

The Scorer will have these bars in his row:  . Now, if the Shooter wants to use a "single shot", the Scorer checks his row, sees that he has a one-unit bar in the last position, and replaces it with a two-unit bar.

Shooter:



Scorer:



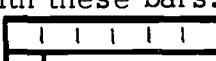
The Shooter needs just one more unit to complete his row and hit the target. If the Shooter makes a single shot however, the Scorer must forbid the shot -- the Scorer does not have a one-unit bar in the last position to remove. In this instance the Shooter and the Scorer must begin again -- this sequence cannot be successfully completed.

Here is a sample game using a six-unit target:



- a) Each pair of players start with these bars:

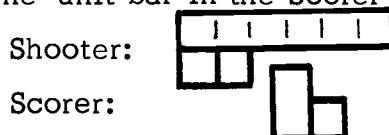
Shooter:



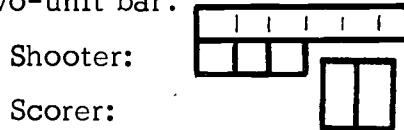
Scorer:



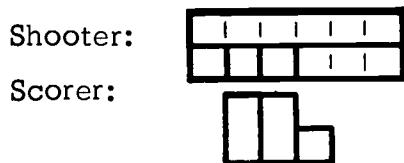
b) The Scorer sees that he cannot allow the Shooter to use the single shot because there is no one-unit bar in the last position of his row. The Shooter must match his shooting row. (The result, in this case, of course, is the same.) The Scorer records this match shot by placing a one-unit bar in the Scorer's row.



c) Now the Shooter decides to use a one-unit bar. The Scorer looks at his row and sees that he does have a one-unit bar in the last position, so he records the shot by removing the one-unit bar and replacing it with a two-unit bar.



d) The Shooter now matches his shooting row. To record this, the Scorer places a one-unit bar beside the others in his row. With this move the Shooter and the Scorer have hit the target.



It may be advantageous to teach the game in the following way:

- 1) The match shot might be taught to the whole class. While the children are practicing this shot on various bars, the teacher can say that she will keep a record of the shots. She will then follow the Scorer's role for scoring.
- 2) Next, the whole class can be asked to exchange roles with the teacher. Have the children keep score while the teacher uses the match shot.
- 3) The children can now be taught the single shot and how it is scored simultaneously.

- 4) Finally, let all of the children be Scorers while the teacher is a Shooter. The Shooter now uses either the match shot or the single shot in trying to hit the target.
- 5) Divide the children into pairs and select which child is to be a Shooter and which is to be the Scorer. The Scorer chooses that target that the Shooter must try to hit. Have the Shooter and the Scorer exchange roles each time a target is hit.

The purpose of the story "Tal's Aching Back" is to show how much simpler life can be after mastery of a set of counting words.

Before reading the story, it might be advisable to sing several counting songs, such as "Ten Little Indians", "One, two, buckle my Shoe", etc.

Discuss the following with the children:

"Do you remember Ugboo? If he wished to tell a friend how many sheep he had taken up to the mountain, what would he have to do?" Answer: he would have to show the friend a stone for each sheep.

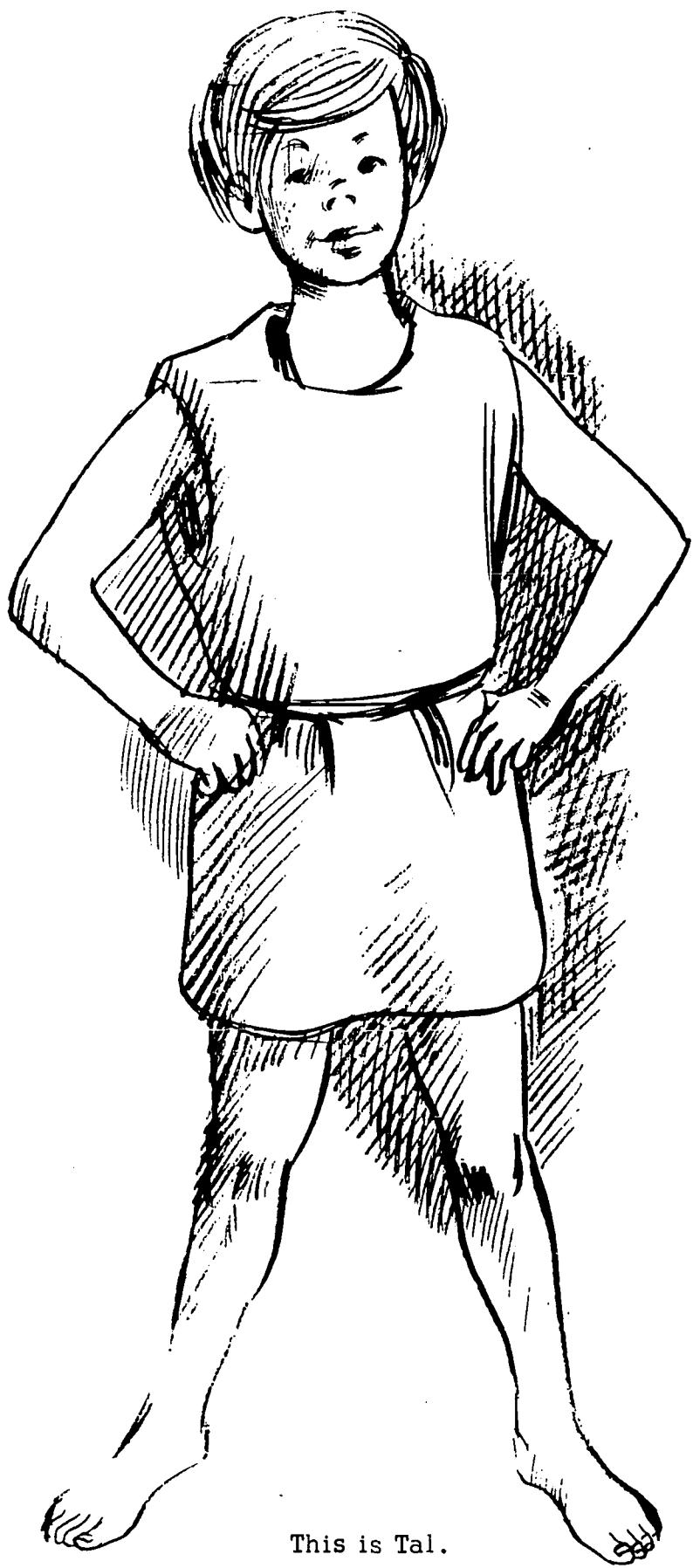
"If you were Ugboo and your mother sent you to the store for some oranges, what would you have to do?" Response: carry along a stone for every orange you were to bring.

# TAL'S Aching Back



Using Words to Represent Numbers

TAL'S ACHING BACK



This is Tal.

## TAL'S ACHING BACK

One evening, David and his father were singing "Ten Little Indians". While they were singing together they set David's toy Indians up in a row. He laughed and laughed when a touch on one sent the whole row tumbling down.

David asked, "Who thought of calling one, 'one', Dad?" Mr. Wright answered, "I'll tell you a story about that if you get into your pajamas and get ready for bed yourself."

David shouted, "Oh, boy, another story! I'll hurry, Dad. I'll be ready in a minute!"

Here's the story David's father told him.

Tal was Ugboo's great, great, great, great, great, great, great, grandson. That means Tal lived many hundreds of years after Ugboo. By this time everyone in Tal's tribe had learned about the great invention of Ugboo. A person would always carry around with him a little sheepskin bag of stones. Whenever he wanted to compare sets that were far away from each other, he would first match one stone with one member in one set. Then he would carry these stones to the other set, and match each of the stones with each member in the other set. In this way it was easy to find out which of two sets had more members than the other.

TAL'S ACHING BACK



Tal's tribe lived by the shore of a lake. They caught fish for food.

## TAL'S ACHING BACK

The people in Tal's tribe lived by the shore of a lake. They caught fish for food. Very often they would have more fish than they needed for themselves.

Besides fishing, the people hunted animals for food. They used bows and arrows. The arrows had sharp stone heads. The right kind of stone for these arrowheads could not be found near the shore of the lake. So when the people in Tal's tribe had extra fish or meat, they would trade with another tribe and receive arrowheads in exchange.

One day Chief Vishtapaka came to Tal, and he said, "We have a lot of extra fish. Put them into a big sheepskin bag, and take them over to the Annuki tribe on the other side of the hill to trade for arrowheads. Be sure that you get one arrowhead for each hunter in our tribe."

TAL'S ACHING BACK



Tal climbed the hill with a bag of fish and a bag of stones.

## TAL'S ACHING BACK

Tal put the fish in a large sheepskin bag. Next he took another bag and into it he put one stone for each hunter in his tribe. There were many hunters in the tribe so there were many stones in the bag. Tal put the bag of fish over one shoulder and the bag of stones over the other shoulder and began climbing up the hill.

It was a hot day. The sun was beating down on Tal's head. By the time he reached the top of the hill, he was quite tired. It was a little easier going down the hill, on the other side. When Tal reached the bottom of the hill, he went to the chief of the Annuki tribe and said, "My Chief, Vishtapaka, has sent me to trade these fish for arrowheads. He wants one arrowhead for each stone in this bag."

TAL'S ACHING BACK



Tal and Chief Darayavush matched arrowheads with stones.

## TAL'S ACHING BACK

Chief Darayavush of the Annuki tribe smiled, then exclaimed, "That is a very good idea! We certainly would like some fish."

Tal emptied his bag of fish. He took the stones from the other bag and placed them neatly on the ground. Chief Darayavush put down one arrowhead for each stone. When the trading was finished, Tal placed the arrowheads in the large bag in which he had had the fish, and shoved the stones back into the other bag. Then Tal began his long climb up the hill again, to get back home to his own tribe.

In the afternoon the sun was even hotter. The two bags on Tal's shoulders seemed heavier than ever. By the time he came down the other side of the hill his back and shoulders felt very sore, indeed. He stumbled into the camp and let the bag of arrowheads and the bag of stones slip down to the ground. "Here are the arrowheads, Chief Vishtapaka," said Tal, wearily. "There is one arrowhead for each hunter in the tribe." The chief was very pleased and gave Tal a friendly clap on his back. "Ouch, my aching back!" cried poor Tal, as he went to find a shady tree to sit under.

## TAL'S ACHING BACK



Ningal felt sorry for Tal and rubbed his aching back and shoulders.

## TAL'S ACHING BACK

As he was resting, his friend, Ningal came to him. "Tal, why are you lying there?" she asked. "Get up and let's have a race."

"Oh, no!" sighed Tal. "I am much too tired. I have been carrying fish and arrowheads and stones all day."

"Why?" asked Ningal.

"Because I had to trade fish for arrowheads."

"But why did you carry stones?" asked Ningal.

"I had to," Tal exclaimed. "I had to have one stone for each hunter in our tribe so that I would know how many hunters there were. I had to get one arrowhead for each hunter."

Ningal felt sorry for Tal, and rubbed his aching back and shoulders.

"There must be some better way to compare sets. There must be some way to do it without carrying around a big bag of stones," thought Ningal.

Tal said, "There is no other way. Everyone uses the wonderful way that Uggabuggabuggabugboo invented. You use one stone for each member in a set and by matching in this way you can compare sets."

"I know," answered Ningal, "but when there are many members in a set, you need many stones. They are very heavy to carry. There must be some other way....."

They sat thinking quietly for a while. How could they match sets and still use something that was easier to carry than a bag of stones?

TAL'S ACHING BACK



"Let's see if my new idea works," said Ningal.

## TAL'S ACHING BACK

Suddenly Ningal jumped up with excitement. "I have a wonderful idea!" she said. "You know how we count one, two, many? Well, instead of 'many', if we could find some more words to carry in our heads it would be so much easier. Then if we wanted to find out how many members there were in a set, we could just begin matching these words with the members in the set. Words are easier to carry than a bag of stones. We could match one word for each member."

Ningal exclaimed, "We all know that we call our fingers one, two, three, four, five, six, seven, eight, nine, and ten. We can't easily forget these names. I'll use my tooth necklace and match one tooth with each of the names, one, two, three, four, five, and six. Now I have a tooth for one, a tooth for two, a tooth for three, a tooth for four, a tooth for five, and a tooth for six. My idea does work! It does!" Ningal was so happy.

"I'll try your idea by matching the same words with these stones," said Tal. "Each time I put down a stone I'll say a word: one, two, three, four, five, six, seven, eight, nine, ten."

"How many do you have?" called Ningal as she flitted around looking for more objects to count.

"I have a stone for one...a stone for two...a stone for three...a stone for four...a stone for five...a stone for six...a stone for seven...a stone for eight...and a stone for nine...and a stone for ten. Whew! I'm all out of breath! Ningal," said Tal, "maybe you can think of a way in which we won't have to say each number word every time we want to tell how many we have."

TAL'S ACHING BACK



"We could use the names of our toes," said Tal.

## TAL'S ACHING BACK

"I'll try," frowned Ningal as she sat down on a big rock. Suddenly Ningal jumped up again with excitement. "Tal, I think I've got it! We don't really have to say all those words every time. The last word we use could be a short way of telling the number of members in a set. We always say the words in the same order anyway, so if I were just to say the word 'six' you would know that I have a tooth for every number name from one through six. Once everyone gets used to using this short-cut, we can simply tell each other how many objects we have. Do you understand?"

"Yes, I do!" exclaimed Tal. "I would just have to use the word 'ten' to tell the number of stones in my set of stones, and you would know that I have one to match with every number word from one to and including ten. This is a terrific discovery. You are a very smart girl! I am proud of you!"

Tal thought for a while, then he jumped up with excitement. "Ningal, I have an idea too! If there were things to match and we used up all the finger names, we could then use the names of our toes. We could say eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, and twenty. We could match these words, one word for each member in a set. The last word we use would tell us how many members there are. But I agree with you, we must all remember the words in the same order; otherwise, we would get mixed up."

TAL'S ACHING BACK



Tal and Ningal ran down to the lake for a swim.

## TAL'S ACHING BACK

Tal was so excited that he did not feel tired any more. He and Ningal ran down to the lake for a swim.

Tal and Ningal explained to the other people in the tribe their new way of comparing sets. When they grew up, Tal and Ningal married. They had many children. They taught their children to match the number words with the members in a set. They called it "counting".

Counting was one of the greatest discoveries ever made. All the other tribes learned counting from Tal's and Ningal's tribe. Life became much easier for everyone when people learned to count.

"That is the end of the story," Mr. Wright said.

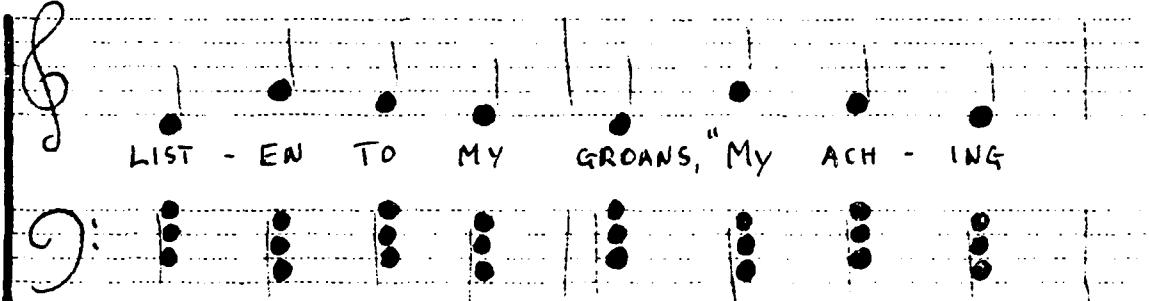
"Thanks, Dad," said David. "We surely are lucky that Tal and Ningal invented counting. It's so much easier to carry words in our heads than stones on our backs."

# TAL'S LAMENT

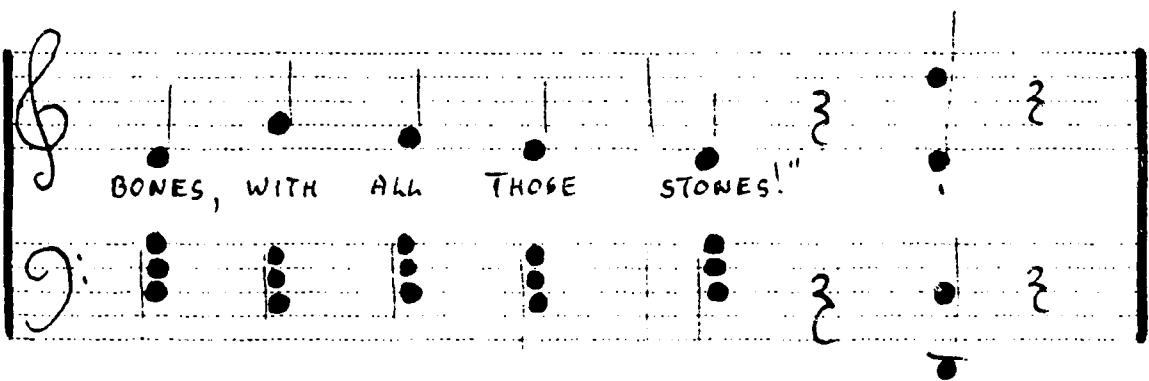


I CLIMB-ING UP THE STEEP HILL-SIDE, THIS HARD WORK I CAN-NOT BIDE

REFRAIN:



LIST - EN TO MY GROANS, "My ACH - ING"



BONES, WITH ALL THOSE STONES!"

2. Correspondence one-to-one,  
Hauling stones is not much fun. Refrain....
3. Carrying these stones all day,  
There must be some better way. Refrain....
4. Count my fingers one, two, three,  
They're always attached to me. Refrain....
5. "Better still," my friend then said,  
"Carry numbers in your head." Refrain....

### Suggested Activities on Counting

1. Follow the story of Tal with a discussion of the following ideas:

Some of the English number words are one, two, three, four, five, six, seven, eight, nine, ten. We must remember these words in this order, and carry them around in our heads. We match them, one at a time in this order, with the members of any set. The last word we use in counting we also use to represent the number of the set. We can always construct a set which matches the counted set by using the word symbols we memorized.

2. Sing counting songs, such as those suggested in the introduction to the story. Practice until each child has the words and the sequence firmly in mind.
3. Place a handful of 8 or 9 counters at one end of a table and choose someone to count them. After the child has finished counting and has announced how many counters he thinks there are, count out a second pile equal in number to the announced result of the first pile. Then have the child match the counters in his pile by one-to-one correspondence with those in the second pile. If he has counted correctly the piles should match exactly.

Repeat the process, choosing a different number for the first pile and a different child to count. When a child miscounts, lead the class to discover that in order to be accurate they need to separate each counter from the group as it is counted. Demonstrate correct counting procedure and compare the methods used by a child who counts correctly and a child who doesn't.

4. Repeat the above activity, this time including counters of several different colors and spreading them on the table. Ask the children to count only the color you designate and then continue as above - counting and matching the second pile. This activity makes it even more imperative that the child separates each counter from the group as it is counted in order to be accurate.
5. Tap your pencil on the table and choose someone to tell you how many taps you made. Begin by giving slow even-paced taps of small numbers. As the children become more accurate in their counting, increase the number of taps and then the speed of the taps. Do not go unreasonably fast. If the class is very good at listening and counting the taps, you might try tapping in an uneven rhythm to see if they are still capable of counting the taps accurately.
6. Place sets of objects on the flannel board. Have children count, first in chorus, then individually. Rearrange objects. Have the children observe that the result of counting is unchanged.  
Have the children separate the set of objects into subsets. Have them count each subset.
7. For practice in making sets with a specified number of members, have a child count aloud a given number of objects and place them on the flannel board, in a cup, or on a desk.  
Tell a child to walk a certain number of steps in a given direction.
8. Play the following team game after dividing the class into two teams. A child from each team comes to the front of the room. They are seated several feet apart, backs to one another.

The first child takes a number of beads, (stones, shells, cards, blocks, or any available object) and describes for the second child how many he has by saying, "I have six beads." (Check to be sure the first child has counted correctly.)

The second child then picks up as many beads as he believes the first child has selected. The teacher asks the class to check by counting. If the second child's counting is correct, his team scores a point. If the second child is incorrect a member of the first team other than the first child may choose the beads.

After a point has been scored in the game, it is the second child's turn to select a set of beads and describe them by number. The other child picks up what he thinks is the appropriate number, and holds them up for the teacher and class to check by counting. Scoring continues as before.

Keep score by tallying on the chalkboard or mounting a cut-out on the flannel board each time a team scores. Have the children determine the winner by matching the tally marks to see which team has more. For variation the children may play this game by two's after they have become adept. A third child could help by checking descriptions and keeping score.

★ 9. Play #8 using other words to count with. For example, the children might use the names of their fingers (thumb, pointer, middle, ring, pinkie), or the days of the week, the alphabet, or the Minnebar colors. (the appropriate Minnebar might then be used as a symbol for the number.) Lead the children to understand that any set of names could be used, but it is easier to understand each other if we use the same set of names every one else uses.

10. Play the following game:

Appoint a child as teller. Give each of the other children one object. Each child in turn comes forward and places his object in a pile before the teller. Each time the teller says, "I have \_\_\_\_\_ objects. You give me 1. Now I have \_\_\_\_\_ objects." He should count each time before he says how many objects he has. Children can take turns being teller. After some experience with the game described a child may come forward and say, "You have \_\_\_\_\_ objects. I give you 1. Now I think you have \_\_\_\_\_ objects. Count and see whether I am right." The teller checks by counting. The game may be varied by giving each child two objects to place in the pile each turn, etc.

★11. The teller in Activity 10 may also give away 1, 2, or 3, at a time. He would say, "I have \_\_\_\_\_ things. I give you \_\_\_\_\_. How many do I have left?" Answer: The recipient must count to check.

12. Barter Store

The storekeeper has a stock of various items. Each child has "wampum" e.g., a set of beads, or counters. Each item has a "price" of so many beads apiece.

A child might say, "I want to trade these beads for 4 toy men. They cost 2 beads; you give me 1 toy man, I give you 2 beads, you give me, etc. Now I have 4 toy men. I gave you \_\_\_\_\_ beads."

★13. Last Child

Two children are chosen to compete to see who can take the last child away from "the circus" (swimming pool, birthday party, or

movie). The purpose is to develop a winning strategy when concepts of one and two are used.

Have all the children except Bill and Mary (children chosen to compete) sit or stand in one area. Tell them that they are at the circus, and are having a very good time! Bill and Mary will see which one can take the last child (class member) home from the circus. They may choose one child at a time each turn, or they may choose two—one child with each hand. The children playing this game alternate turns.

Children may play this game in pairs using objects such as blocks, toys, marbles, beads, etc., instead of children.

Note:

Watch for initial discovery of the winning position, and bring out the idea of winning numbers. When there are four children left, the person whose turn it is (player-A) could take just one child. Then regardless of whether the other player takes one or two it will not matter because player A will win. If the children do not discover this strategy, they should not be told. Encourage them to continue the game at activity time, at home, etc., until they discover it for themselves. They may not learn this strategy until a later time.

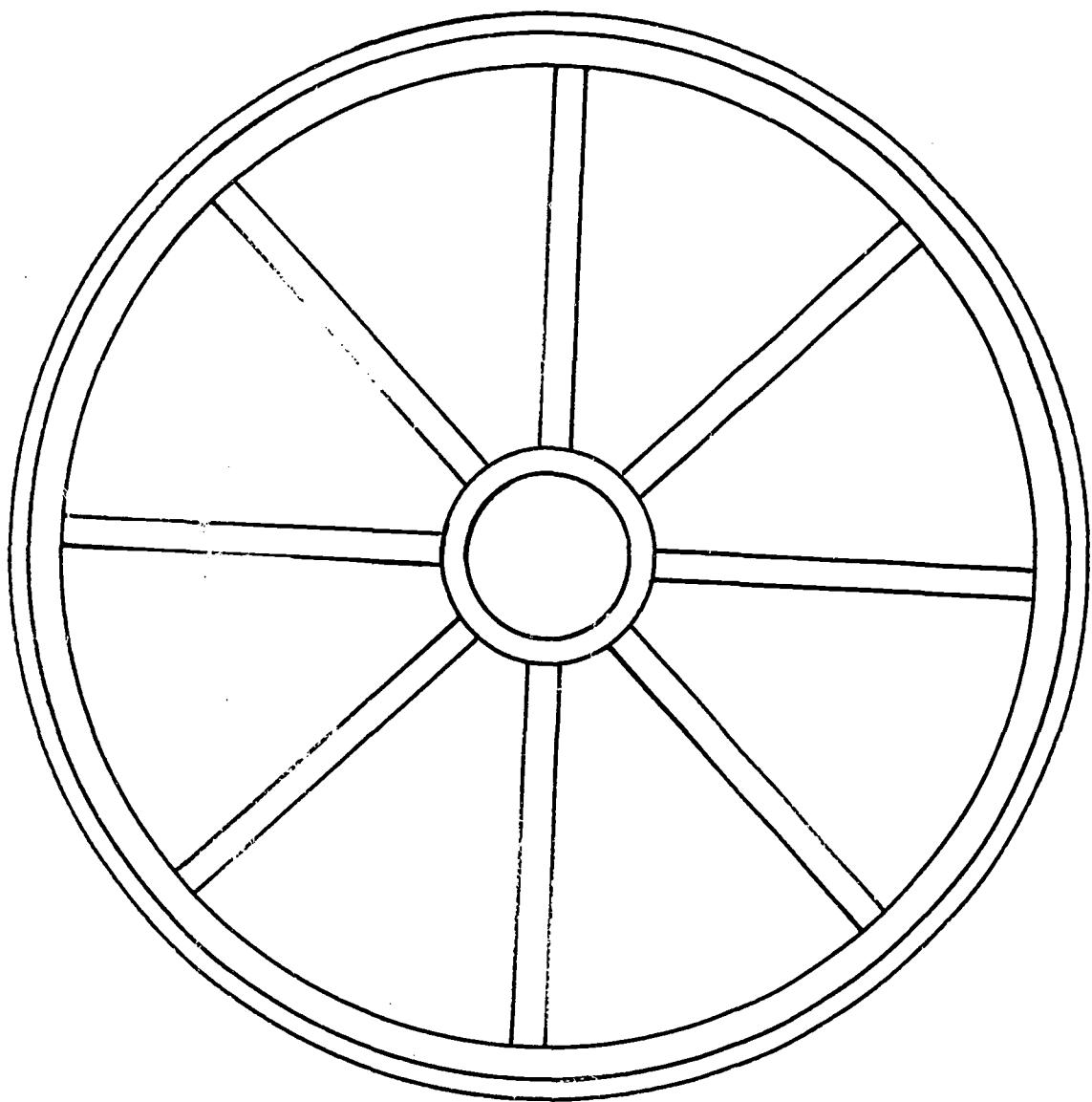
14. Show the children a red, a white, and a blue card. Put them in a row with the red first, then white, then blue. Ask for a volunteer to put them in a different order. Have the class find all the different ways to arrange the cards in a row. Record these different ways on the chalkboard. Have a child tell how many different ways were found all together.

Use three blocks or beads to help the children discover that changing the objects to be arranged does not change the number of arrangements.

## Commentary on Worksheet 9

Distribute the worksheet and ask the children to count the spokes on the wheel. After they have done this, ask them to make an equal number of tally marks on the bottom of the worksheet, one for each spoke. Compare the answers of different children in the class.

Next, compare the methods of counting of those who counted correctly and those who miscounted. Help the class to discover the need for marking off the starting point or separating in some way the spokes already counted.



Count the spokes on the wheel.

Make a tally mark below the wheel for each spoke.

## More Activities Using Minnebars

At this time the bars should be referred to by their number names rather than by color.

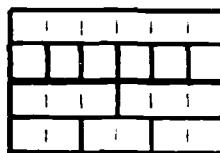
1. Review activities  $\star 2$ ,  $\star 3$ , and  $\star 7$  from pages 22-23 using the number names.

Play these activities without the color showing.

★ 2. Ask the children to match the six-unit bar, for example, using only the three-unit bars. Next, ask them to match the same bar using only the two-unit bars. Point out that in both of these matches, the children matched the "target" using only bars of the same length.

Now select another even unit bar and ask the children if they can match it using all bars of the same length. After one solution has been found ask for another solution. For the children who are slow to find a second solution, ask how many bars are needed to make the match, and how long each bar is.

Help the children to discover that if a bar can be matched with, for example, 2 three-unit bars, it can also be matched with 3 two-unit bars. To aid in illustrating this, it may be helpful to have the children place their solutions directly under each other as shown.



3. Have the children classify the bars by how many ways they can be matched using only shorter bars of equal length. (The two, five,

and seven-unit bars can be matched in the fewest ways, and the six, eight, and ten-unit bars can be matched in the most ways.)

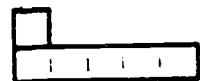
4. Have the children use only the one, two, and three-unit bars to match any of the other, larger bars. To make the activity more interesting, the smaller bars may be called "shots" while the bars to be matched can be the "targets". See who can hit the target in the least number of shots (it may take only one shot). The children may wish to keep track of their shots by using tally marks.
5. Challenge the children to investigate whether they could play activity 4 using only the one, two, and four-unit bars as shots, that is, can all the bars be hit using only these three lengths?

This investigation may be carried further to see if the activity is possible using only the two-unit and the five-unit bars as shots. (In this case, all but the one-unit bar and the three-unit bar can be "hit".)

At this point someone may think of "overshooting" and coming back to hit the target. For example, the one-unit bar might be hit in three shots by using the five-unit bar as one shot, and then coming back with 2 two-unit bars:

Target: 

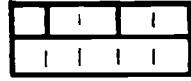
Shot #1



Shot #2



Shot #3



6. Let the children make up their own rules, specifying what bars can be used for shots and whether overshooting is allowed, not allowed, or required. They might then investigate which targets can or cannot be hit.

7. The children might play in pairs or compete in teams. One child (or -- team) would make up the rules and specify which target is to be used. In other words, this child's duties would be to pick the bar that is to be matched (set up the target) and state the rules to be used in matching it by specifying what (or how many) bars may be used to match (or hit) the target, and also by specifying whether overshooting is allowed, not allowed, or required.

To add interest, this child might be called the "problem poser" or perhaps the "targeteer", while the other child, who must obey the rules and hit the target, may be called the "problem solver" or the "shooter".

If the solver can obey the rules and still hit the target, he scores a point. If the solver cannot solve the problem, he says, "I cannot hit the target." If the poser can then hit the target -- within the restrictions of the rules -- he will score a point for himself. If, however, the poser cannot hit the target either, the solver is awarded the point. The children may take turns being posers or solvers.

8. Activity 7 might be more simply played under the general rule that the target should be hit using as few bars as possible. In other words, if the poser could complete the match using fewer bars than the solver, then the poser would score a point.
9. Review the Shooter's role in Introduction to Mathematical Golf (p. 25). In this game the Shooter was asked to hit a target by perfectly matching a target Minnebar with several smaller bars. In order to hit this target, the Shooter was given a choice between

two kinds of shots:

1) The "Match Shot"

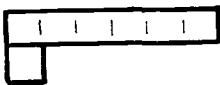
The shooter could find a bar to match the bars in his shooting row and place that bar at the end of the row.

2) The "Single Shot"

The Shooter could place a one-unit bar at the end of his shooting row.

Each game was started with the Shooter's bars in this position:

Target:

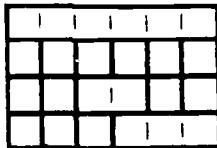


Shooting row:

In Mathematical Golf, the Shooter was occasionally restricted in his choice of moves by a Scorer. In this variation, however, allow the children to make either move whenever they wish.

Challenge the children to see how many different patterns of bars they can use to hit the target using the Shooter's shots.

It may be helpful to line up the different patterns of shots in rows under the target as shown:



After the children have found as many different patterns of bars as they can, ask them to count the bars (not the units) in each row to determine the least number of bars needed to hit the target.

Next, ask them to find how many bars are needed to hit the target in the Mathematical Golf game when there is a scorer to restrict their moves. A review game of Mathematical Golf using this target may be necessary at this point. Someone may discover that the Scorer's restrictions always produce a solution which uses the least number of bars.

# Ruth Ann

at the

# U.N.



Written Numerals

RUTH ANN AT THE UNITED NATIONS



This is Ruth Ann Wright.

## RUTH ANN AT THE UNITED NATIONS

It is Wednesday, two o'clock in the afternoon. The sun is shining in a blue-white sky. The Wright household is in a state of excitement, waiting - waiting for someone. Just when it seems as though David cannot wait another minute, a shiny green car stops in front of the house and from it jumps a pretty, bright-eyed girl. She throws a kiss to the lady at the steering wheel and says breathlessly, "Goodbye, and thank you so much for meeting me at the station. I'll see you tomorrow. Goodbye."

Grandma and Grandpa Gay's "Goodbyes" are still ringing in the air as the girl dashes into the house calling happily, "Bonjour (Bawn-JOOR)! Shalom (sha-LOM)! God Dag (gou-DAW)!"

David runs to her, shouting, "Hi! Ruth Ann." Then he stops. "Just what are you saying?" he asks in surprise.

"My goodness," laughs Mrs. Wright as she hugs and kisses the girl. "Say 'Hello' in English, and tell us about your trip. But first, would you like a sandwich and a glass of milk?"

"Oh, yes please. I'd like that!" smiles Ruth Ann.

RUTH ANN AT THE UNITED NATIONS



Almost before she has finished her snack, a group of children burst in upon her.

## RUTH ANN AT THE UNITED NATIONS

Almost before she has finished her snack, a group of children burst in upon her.

"Hi, Ruth Ann!"

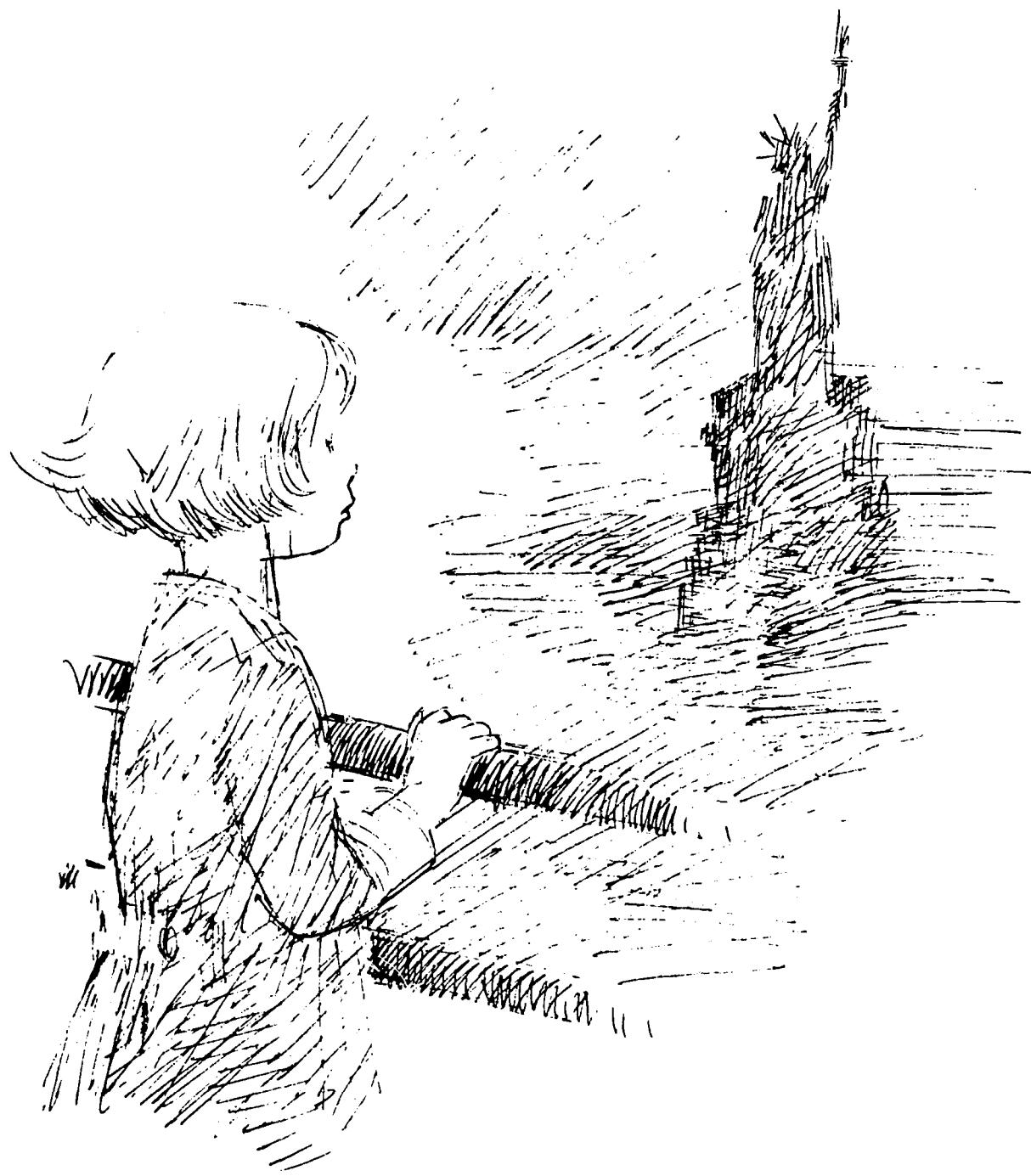
"Hi, Ruth Ann!"

"Hello, Ruth Ann."

"Hello, Ruth Ann!" ring the voices of her little friends. "Tell us about your trip to New York. Please tell us!"

"Well, starts Ruth Ann, "Grandma and Grandpa Gay were ever, ever so nice to me. They bought me new clothes and books, and they took me to the ballet.

RUTH ANN AT THE UNITED NATIONS



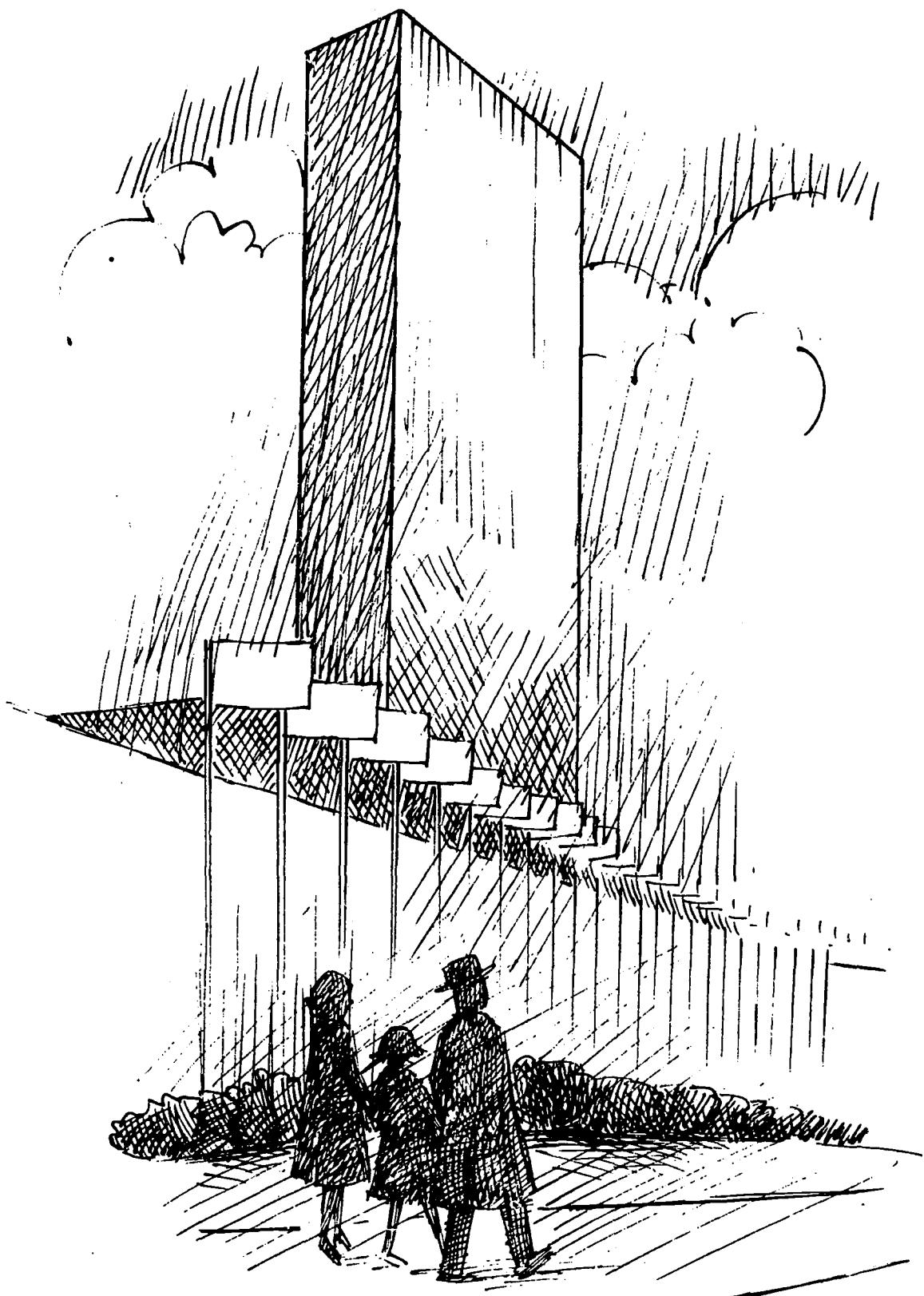
"As I looked south, I saw the Statue of Liberty on an island in the harbor."

## RUTH ANN AT THE UNITED NATIONS

"I went to the top of the Empire State Building which is the tallest building in New York City; the cars looked like tiny toy cars when I looked down at them from way up there.

Then, as I looked south, I saw the Statue of Liberty on an island in the harbor. Lady Liberty was holding her torch up high welcoming the people coming to America."

RUTH ANN AT THE UNITED NATIONS



Ruth Ann went to the United Nations Building.

## RUTH ANN AT THE UNITED NATIONS

"But I had the most fun at the United Nations....." sighs Ruth Ann, happily.

"Tell us about the United Nations. Tell us what happened," chorus the children.

"Well, on Manhattan Island, right near the East River, there is a tall building. It is called the United Nations Building. It is made of marble and limestone and has many, many windows. Inside there are people from many different countries."

"But what do they do there?"

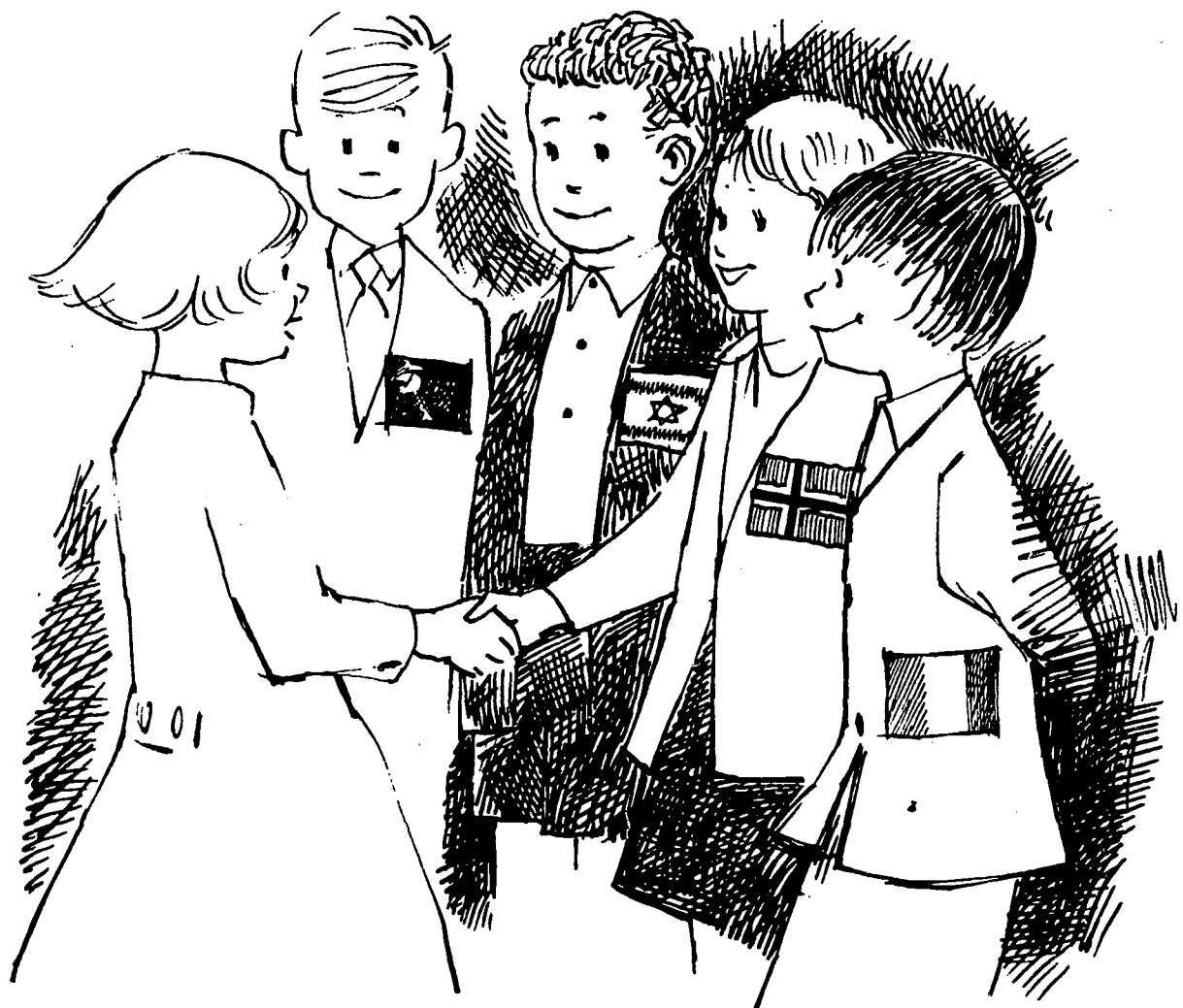
"Grandpa said there are many ideas about which these people do not agree. At the United Nations Building they are learning to talk about these ideas, rather than to fight about them."

"Are they going to become good friends and not have any more wars?"

"I don't really know. All the people were still busy talking when I left, so I don't really know what will happen," sighs Ruth Ann sadly.

"Oh, but something wonderful did happen and I learned that all kinds of children can be good friends!"

## RUTH ANN AT THE UNITED NATIONS



At the United Nations Building, Ruth Ann met children from other countries.

## RUTH ANN AT THE UNITED NATIONS

"I met some girls and boys who were waiting for their parents. We became friends, and we decided not to have any fights or quarrels or wars." says Ruth Ann, now very happy and very proud. "We learned to be friends even though it was hard for us to talk to each other."

"What do you mean? Couldn't you just open your mouth and talk?"

"Yes, but people from different countries use different words. A Frenchman says 'bonjour' instead of 'hello'."

"Oh, so that's what you were saying when you came in! But how could you become friends if you couldn't talk to each other?"

"Sometimes, I could tell what they wanted to say by looking at their faces. They would smile, nod their heads for 'yes,' and do you know what - - -? They would laugh and cry, just as we do!"

"Yes, but what could you say to each other?" asks David a little impatiently.

RUTH ANN AT THE UNITED NATIONS



"When Yoshiko held up the card with the numeral 2 we each said what the number was called in our own language."

## RUTH ANN AT THE UNITED NATIONS

"We started with numbers, because the number ideas are the same all over the world. Even though the number words are different, they mean the same to everyone. There was a lovely Japanese lady called Yoshiko who had a pack of cards with numerals on them. When she held up a card we did some things."

"What sort of things, Ruth Ann?"

"When she held up the card with the numeral 2 we each said what the number was called in our own language."

"What did you say Ruth Ann?"

"Two."

"What did the others say?"

"The girl from France said 'deux' (DUH), the boy from Israel said 'shnahyeem' (SHNAH-yeem), the boy from Russia said 'dva' (DVA), and the girl from Norway said 'to' (to). The funny thing was, that though they each called the numeral on the card a different name, they all knew it meant the same thing."

"We don't understand what you mean, Ruth Ann," wailed the Cooper twins.

"Well, let me finish and you'll get the idea. When the lady held up the card with two on it, she held up two fingers, and all of us held up two fingers to show how many two is. When she held up the card with five on

RUTH ANN AT THE UNITED NATIONS



"They all knew that the numeral 5 meant five."

## RUTH ANN AT THE UNITED NATIONS

it, the girl from Norway said 'fem' (FEM) and held up five fingers!"

"What did the boy from Russia say?"

"He said 'pyat' (PYAT) and guess what? He held up five fingers!"

"The boy from Israel, what did he do?"

"He said "khahmeesheh" (kha-mee-SHAH) and also held up five fingers."

"What about the girl from France?"

"She said 'cinq' (SANK).

"Did she hold up five fingers also?"

"Yes, she held up five fingers when she saw the card with the numeral five on it."

"You mean to say they each held up five fingers? They all knew that the numeral five meant five?" asks David very much surprised.

"Yes! Really and truly!"

"That's amazing!" thinks David aloud. "They all use the same numerals as we do, but they have different number words from ours."

"That's almost right. Nearly everyone in the world recognizes the same written names for numbers, but many people in Asian countries also use different written symbols."

"I don't understand. Can you show us some of these different symbols?" asks one of David's friends.

I can't remember many, but I can show you a few," says David's sister.

"The Arabs write '٢' for two, and '٣' for three, and '٨' for eight."

As the friends are looking excitedly at the strange numerals, Ruth Ann sees that David is sad. "Oh, come on, David, don't look so unhappy.

RUTH ANN AT THE UNITED NATIONS



"Grandpa Gay said 'three' and jumped three big kangaroo jumps!"

## RUTH ANN AT THE UNITED NATIONS

Grandpa and Grandma Gay said it will be your turn to go to New York sometime soon. Cheer up and I'll tell you something that will make you laugh. When the lady held up a card with the numeral 3 on it, Grandpa Gay did something so funny that the lady from Japan and the girls and boys and I just laughed and laughed--in English. I mean they laughed just as we do."

"Well, what did Grandpa do?" asks David, cheering up a little.

"When this lady held up the card with three on it, Grandpa Gay said 'three' and jumped three big kangaroo jumps!"

"Did he really do that?"

"Yes, he really did! Well, after that, when it was the Israeli boy's turn he said 'shloshah' (shlo-SHAH) and jumped three big kangaroo jumps."

"The girl from France, did she know what to do?"

"You bet! She said 'trois' (TRWAH) and gave three jumps. Do you know how many jumps the Russian boy gave after he said 'tree' (tree)?"

"Did he give three jumps?" asks Margaret.

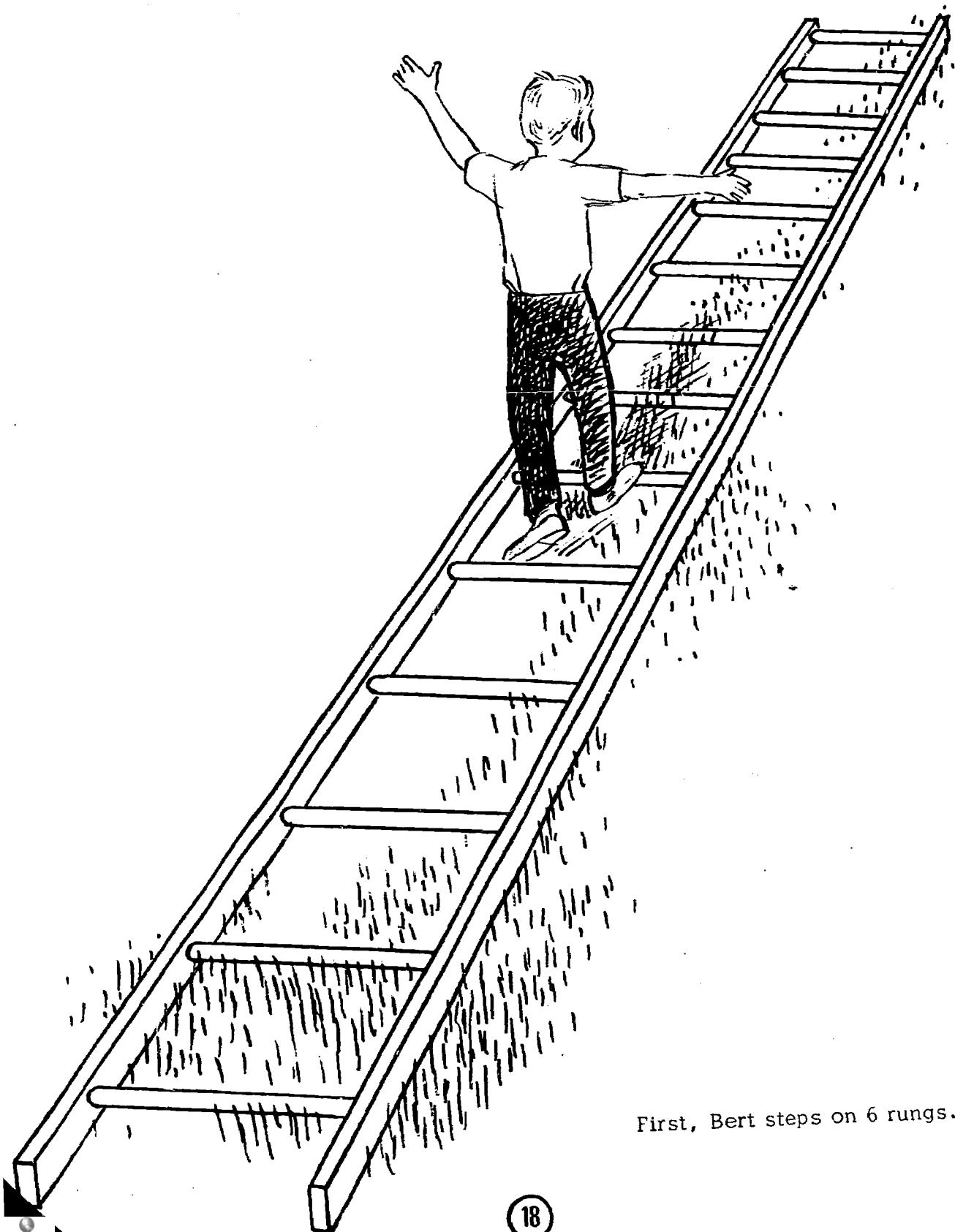
"Yes, because he knew it was three. When the lady held up the card with eight, the Norwegian girl clapped her hands eight times; when the seven was held up I hopped on one leg seven times. It was so much fun."

"I have a set of cards with numerals in my room," starts David hopefully. "Would you like to play this game with us, Ruth Ann, please?"

"I'd love to," smiles Ruth Ann, and feeling very important adds, "but let's play outside in the fresh air. It's much healthier, and there may be more things we can do outside the house than inside."

Ruth Ann sees a long ladder lying on the ground with ever so many

RUTH ANN AT THE UNITED NATIONS



First, Bert steps on 6 rungs.

## RUTH ANN AT THE UNITED NATIONS

rungs on it. (The steps on a ladder are called rungs.) Suddenly an idea comes. Ruth Ann pretends she cannot speak English, so without saying even one word, she holds up the card with the 6 on it. The boys and girls get the idea.

Show the children the appropriate numeral.

First, Bert steps on six rungs. Then each of the friends steps on six rungs.

Do you know how many rungs they walk on for the card with the numeral 1? How many rungs do they walk on when Ruth Ann shows the card with the numeral 9?

Mrs. Wright, who has been watching this number game for some time now, comes to Ruth Ann and with a special look on her face asks, "May I have the numeral cards for a minute please?"

"Certainly, Mother," says Ruth Ann wondering what her mother's going to do with them.

Mrs. Wright holds up the numeral 8, and says, "All girls and boys please take this number of giant steps toward the kitchen!" Do you know how many steps the girls and boys take?

"Now everyone take this number of hops on your right foot," says Mrs. Wright holding up the card with the numeral 6 on it. How many hops do you suppose the children take? Mrs. Wright holds up the card with the numeral 9 on it. "Please take this number of hops on your left foot." Do the children take nine hops? They do, and they find themselves right by the kitchen door.... "Everyone into the kitchen with this number of steps,"

RUTH ANN AT THE UNITED NATIONS



"On, Daddy, I've missed you so much! But I had so much fun!"

## RUTH ANN AT THE UNITED NATIONS

says Mrs. Wright holding up the numeral 7.

What do you think they find on the table? They find delicious cold milk, cookies and apples. There is a glass of milk for David, Ruth Ann, Margaret, Bill, Bert, and Julia. Can you tell how many glasses of milk there are on the table?

There is a cookie for each child. How many cookies are there? There is also one apple for each child. Do you know how many apples there are on the table?

When the snacks are eaten, Mrs. Wright says to Ruth Ann, "Will you take this number of steps toward the living room door, please." She is holding up the card with the numeral 5 on it and on her face there is again that special look which means a surprise is coming up. Don't you wonder what it might be?

Ruth Ann opens the door. "Daddy! Daddy! Oh! Daddy!" she cries happily, hugging and kissing Mr. Wright who has just returned from work. "Oh, Daddy, I've missed you so much! But I had so much fun! It was all so exciting! I have a present that Grandpa, Grandma Gay and I bought for David. We bought David a large globe of the world. I'll be able to point out the countries, where the children at the United Nations come from. Oh, Daddy, do you think he'll like the present?"

Mr. Wright thinks for a moment, smiles and says, "Ruth Ann, that globe will probably be one of the finest gifts David has ever had. I know he will be very happy with it."

Spanish

uno	OO-no
dos	DOHSS
tres	TRAYSS
cuatro	KWAH-troh
cinco	SEEN-koh
seis	SAYS
siete	SYAY-tay
ocho	O-cho
nueve	NWEV-ay
diez	DYES

French

1	un	UNG
2	deux	DUH
3	trois	TRWAH
4	quatre	KAHTR
5	cinq	SANK
6	six	SEESS
7	sept	SET
8	huit	WEET
9	neuf	NUF
10	dix	DEES

Russian

Одн	oh-DEEN
Дв	DVA
Трн	TREE
Четыре	chye-tir' ye
Пят	pyat'
Шест	shyest'
сем	syem'
Восем	vo syem'
Девят	dye' vyat'
Десят	dye' syat'

Norwegian

1	en	AYN
2	to	TOH
3	tre	TRAY
4	fire	FEE-reh
5	fem	FEM
6	seks	SEKS
7	sju	SHOO
8	atte	AWT-TEH
9	ni	NEE
10	ti	TEE

HebrewGerman

<b>אַחַת</b>	a-CHAT	1	eins	INS
<b>שְׁתִים</b>	sh-TA-yim	2	zwei	TSVI
<b>שְׁלַשׁ</b>	sh-LOSH	3	drei	DRI
<b>אַרְבָּעַ</b>	ar-BAH	4	vier	FEER
<b>חֲמָשׁ</b>	cha-MAISH	5	funf	FEWNF
<b>שָׁשׁ</b>	shaish	6	sechs	ZEKS
<b>שְׁבַע</b>	SHE-vah	7	sieben	ZEE-ben
<b>שְׁמֹנֶה</b>	shmoe-NEH	8	acht	AHKT
<b>טְשַׁע</b>	TAI-shah	9	neun	NOYN
<b>עֶשֶׂר</b>	EH-ser	10	zehn	TSANE

ItalianJapanese

uno	OO-no	1	一	ichi	ee-CHEE
due	DOO-ay	2	二	ni	NEE
tre	TRAY	3	三	san	SAHN
quattro	KWAHT-tro	4	四	shi	SHEE
cinque	CHEEN-kway	5	五	go	GO
sei	SAY	6	六	roku	ro-KOO
sette	SET-tay	7	七	shichi	shee-CHEE
otto	OHT-to	8	八	hachi	ha-CHEE
nove	NO-vay	9	九	ku	KOO
dieci	DYAY-chee	10	十	ju	JOO

## Activities on Spoken and Written Numerals

1. Discuss with the children the idea that written numerals mean the same to all people of the world. Number words vary from language to language, although the ideas are the same for all. A request for two cups of coffee would bring forth the same number of cups, regardless of the language in which it was made.
2. Have children assume roles of children from other lands, as in the story "Ruth Ann at the United Nations", and play some of the same games. List number words to ten in various languages. (See Fredrick Bodmer, The Loom of Language, Norton: 1944.)
3. Have some of the children invent a language of their own and teach the rest of the class the "new" number words from one to ten. Pose the question: "Even if these new number words are better than 'one', 'two', etc., why can't we use them in everyday life?" (The new words would be useless unless everyone else agreed to use them too.)
4. Introduce written numerals by discussing the importance of having symbols to represent number. Point out examples, like the following:

"If you wanted to write to your grandmother and tell her you have moved to a new house, you would need number symbols to tell her where you now live."

"When you want to telephone a friend, you need to have the number symbols in order to dial the right number."

"You've probably noticed that television sets have number symbols for each channel so that you can find your favorite TV show."

"The written symbols that we use for numbers look like this:

1, 2, 3, 4, 5, 6, 7, 8, 9

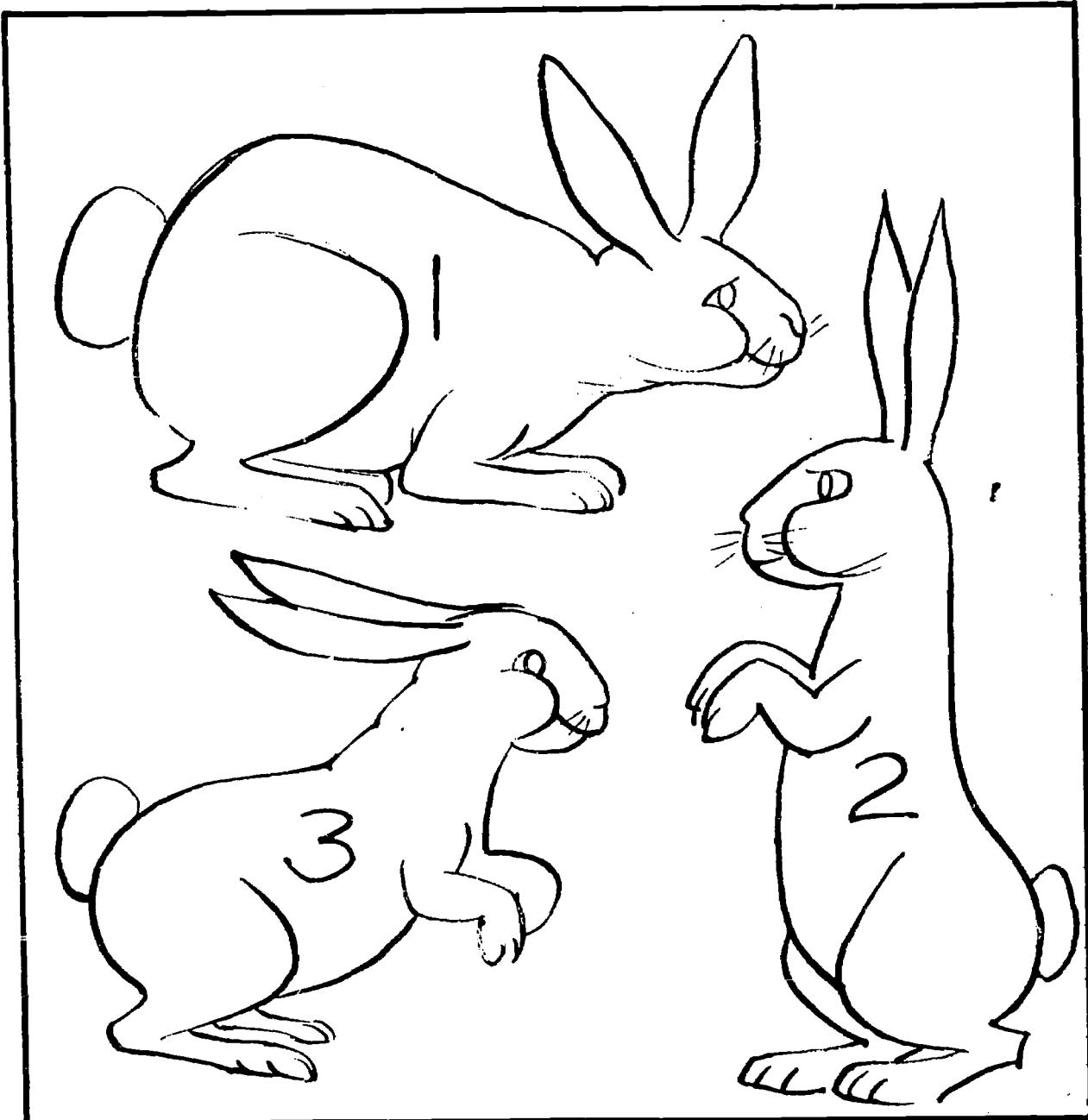
(Write on chalkboard or present numerals on flannel board.)

5. If printed cards with numerals on them are available, the children could practice reciting the word corresponding to the symbol and putting the cards in order. These activities could be individual or competing teams could be formed.
6. Ask the children to think of other symbols they know. Examples: traffic signs, names of foods as symbols for the actual food, numerals as symbols of number ideas, etc.
7. Each child should have the opportunity, either singly or with a group, to use numeral cards and counters at his desk. This might be done in conjunction with teacher-directed exercises such as:  
"Put as many counters on your desk as you have toes."  
"Put the correct numeral beside the counters"  
Have the children take turns making up similar exercises.
8. Prepare two sets of paper cups with the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (T) on the side. (20 in all) Have ready at least 55 counters. These may be beads, flat toothpicks, etc.  
Individually, have the children place as many counters in the one set of cups as are indicated by the numeral on the side. The second set of cups should have the correct number of counters in each cup so the child can compare counters on a one-to-one basis and check his own work.

9. Play the game "What's Missing?" Use the numeral cards 0 through 10. Stand them in order on the chalk ledge while one pupil leaves the room. Then select another child to remove one card from the ledge. At this point the first child is called back into the room to determine which numeral is missing. If he cannot, he chooses a helper. After selecting the appropriate number, the first child chooses the next pupil to leave the room.
10. "Scramble" This game is played like "What's Missing?" This time, however, the cards are mixed up on the chalk ledge and the person who returns to the room is to quickly put the numerals in order.
11. Combine the two games "What's Missing?" and "Scramble" by first sending a child out of the room and then mixing the order of the cards on the chalk ledge and removing one numeral card. When the child returns he must determine which card is missing. If there is difficulty, ask him to first put the cards in order and then tell which is missing.
12. Each of these games can be varied by having 10 children wear the numerals or hold them. Then instead of removing a card, a child would be removed instead. In like manner the line of 10 children would scramble their order themselves. Whenever the child is guessing which person or numeral is missing, he must always identify the missing person by numeral rather than name.

Commentary on Worksheets 10, 11, ★ 12, 13, 14, ★ 15, ★ 16

Worksheets 10 - 16 involve recognition of written numerals. Directions are on the worksheets and are self-explanatory.



3 rabbits

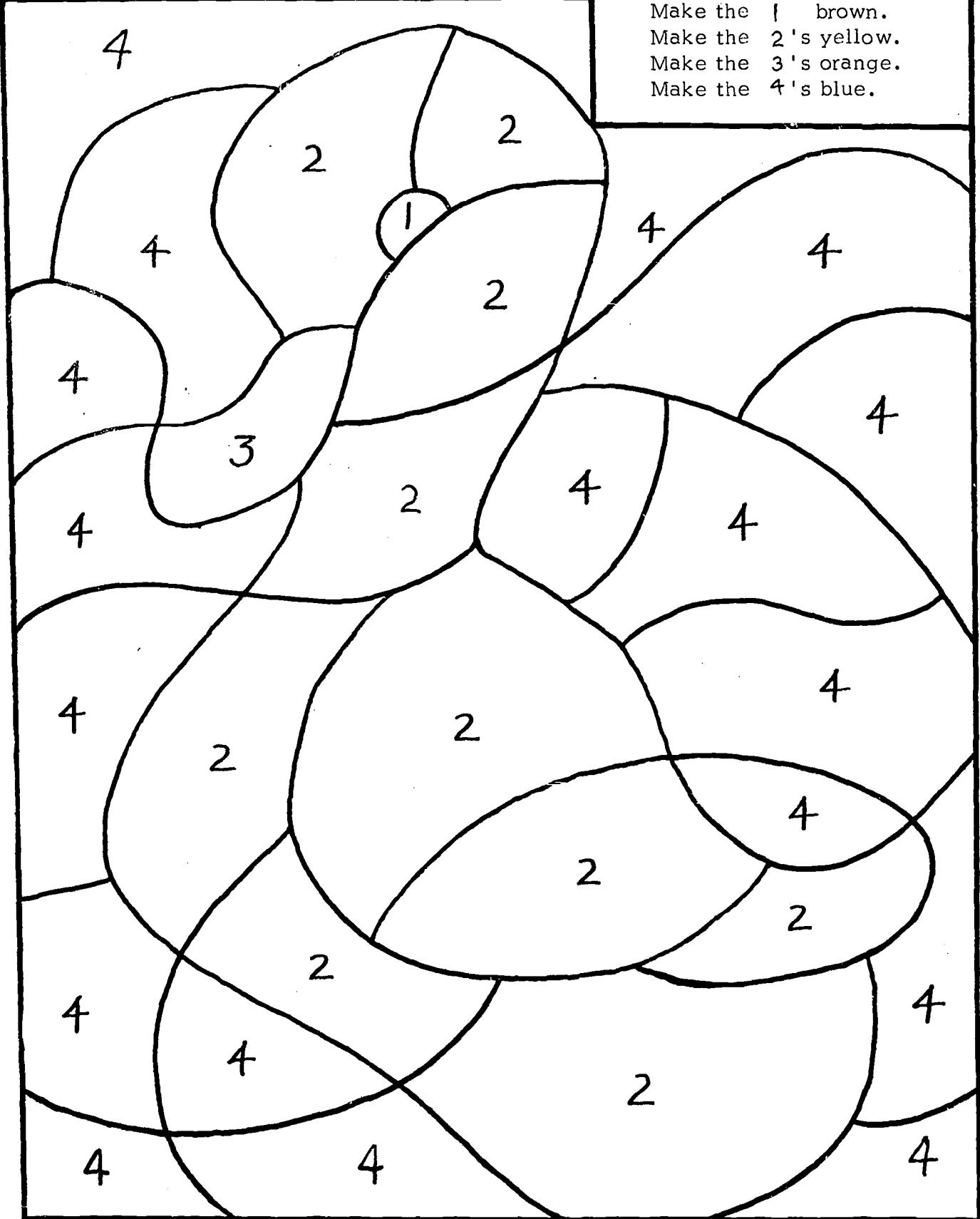
three rabbits

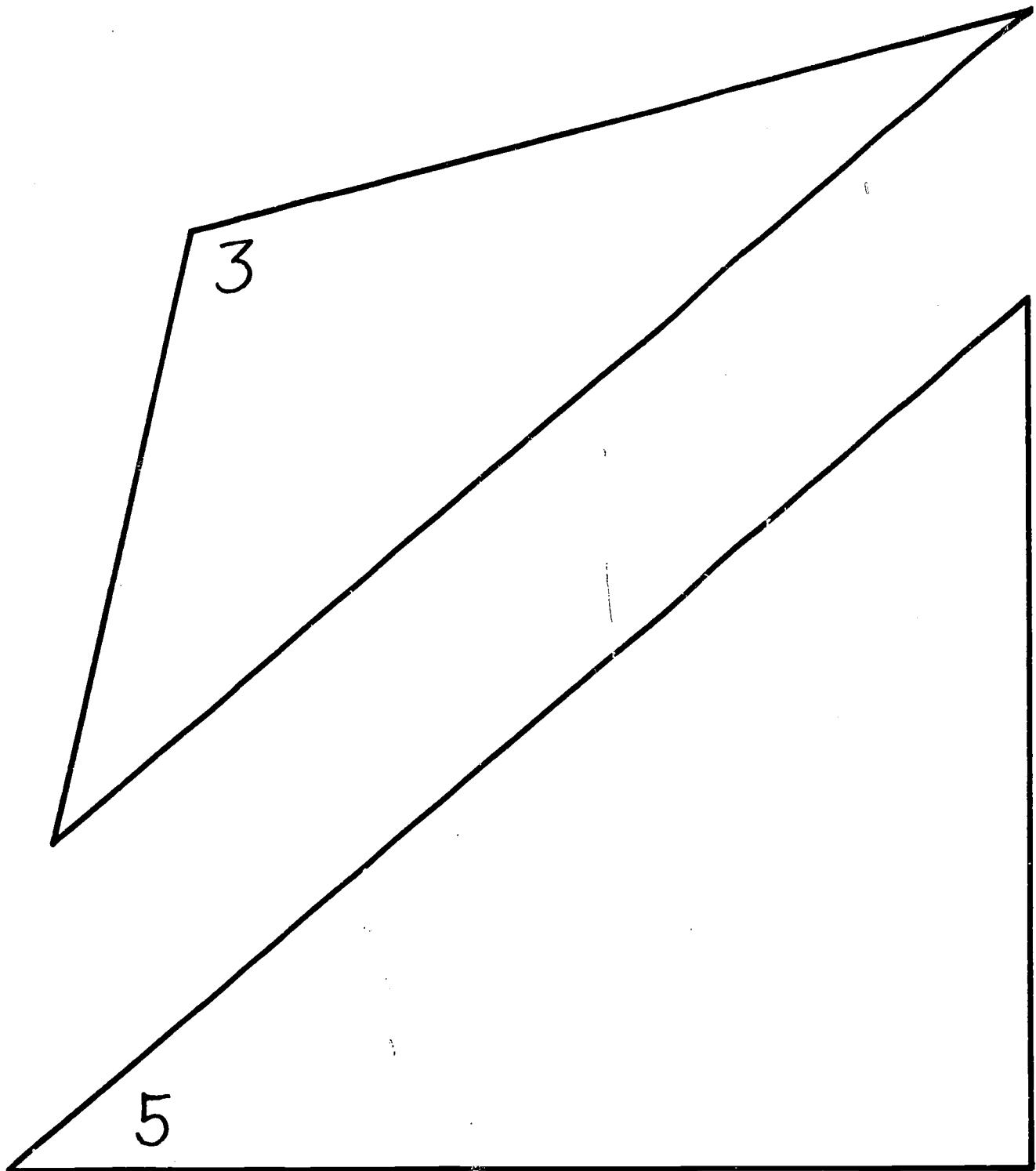
Color rabbit 1 yellow.

Color rabbit 2 orange.

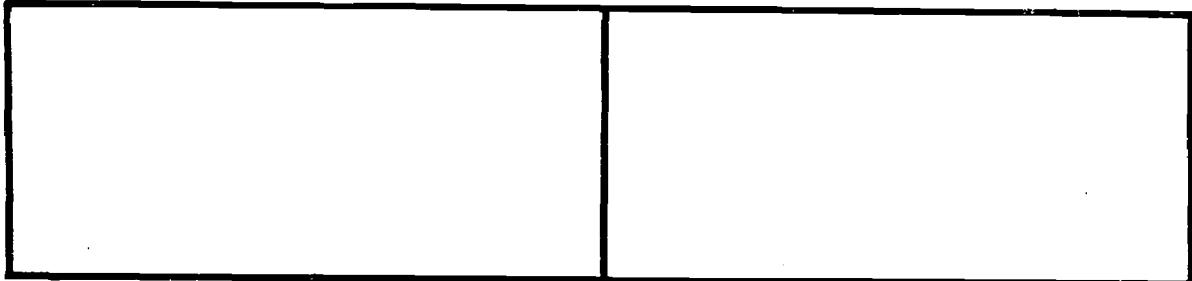
Color rabbit 3 green.

Make the 1 brown.  
Make the 2's yellow.  
Make the 3's orange.  
Make the 4's blue.



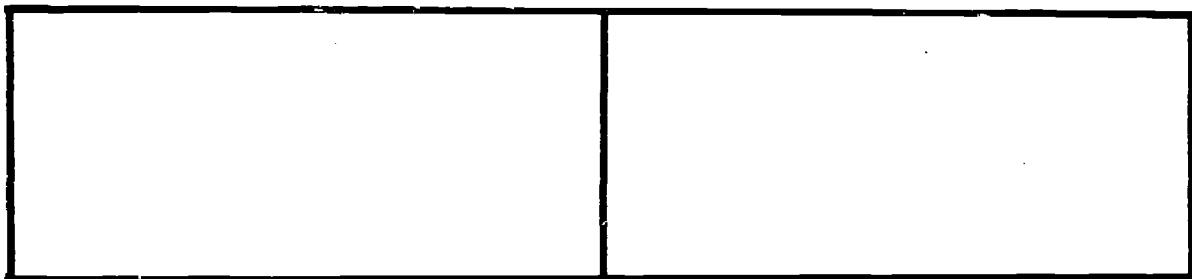


In each triangle draw a set that has as many members  
as the numeral inside the triangle says.



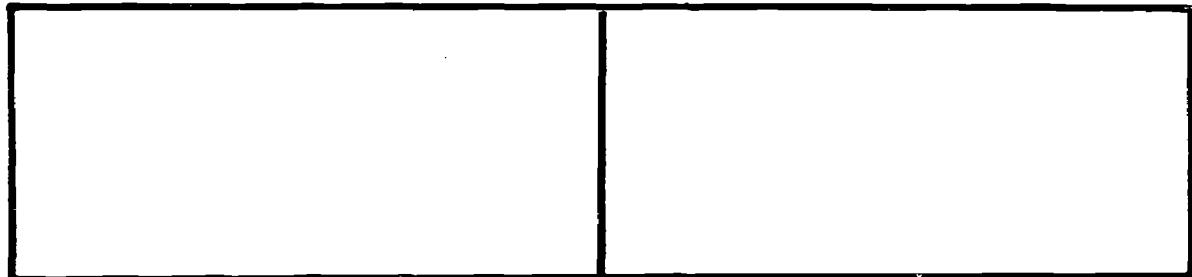
2

1



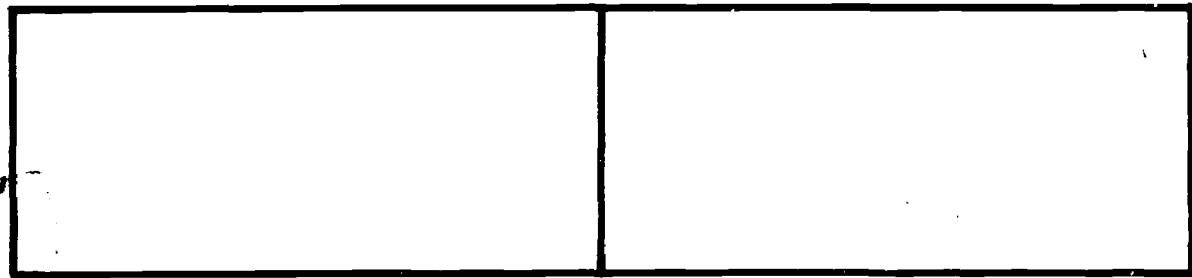
0

4



6

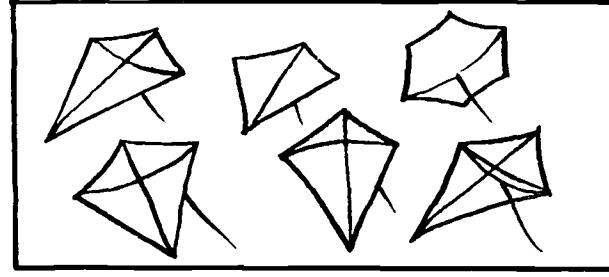
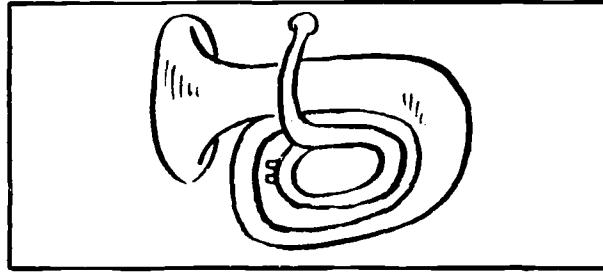
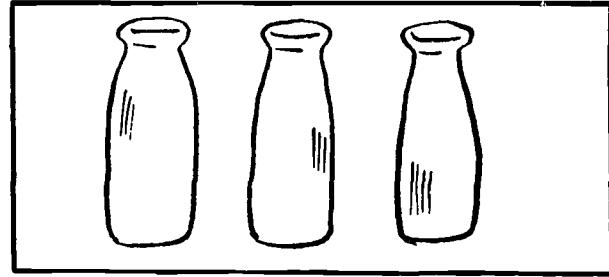
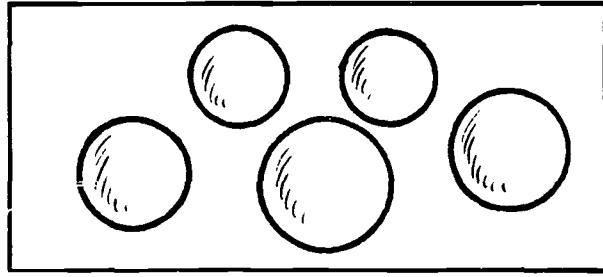
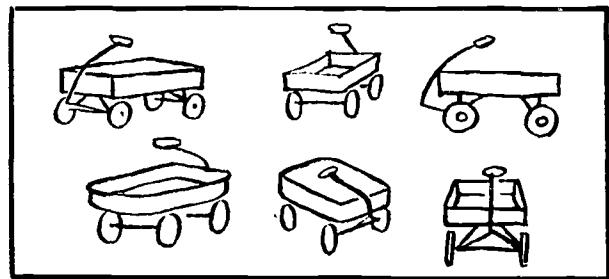
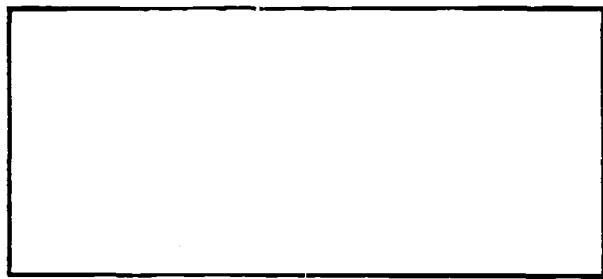
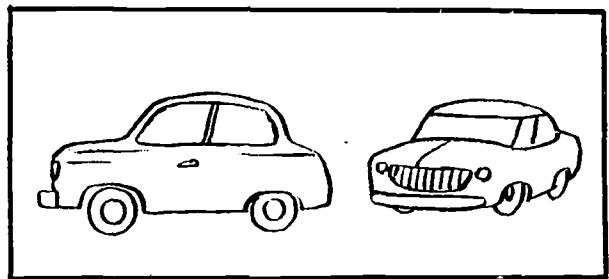
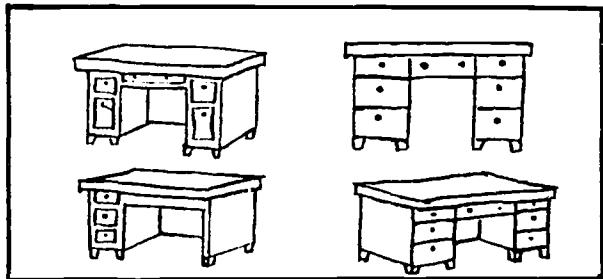
3



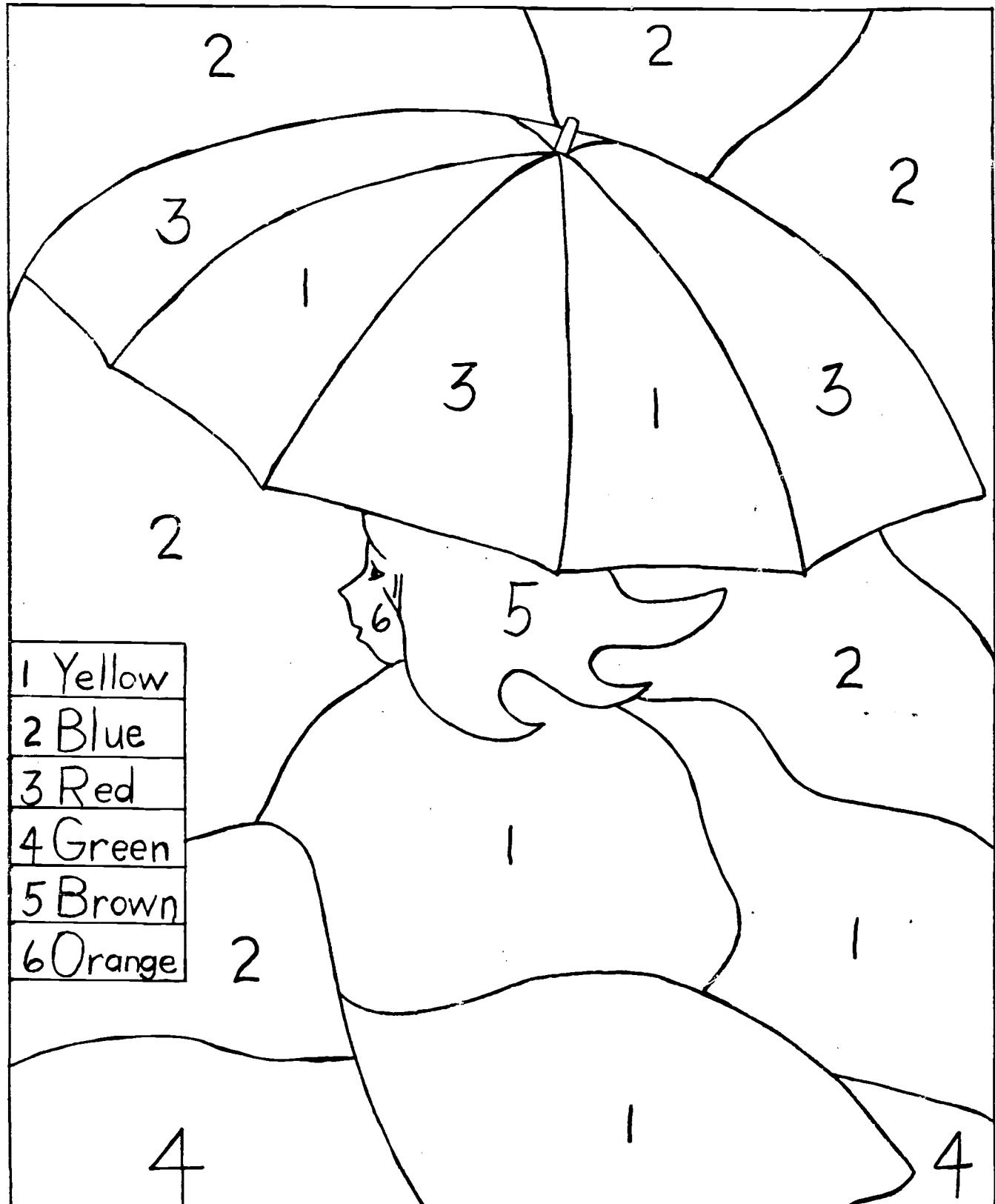
5

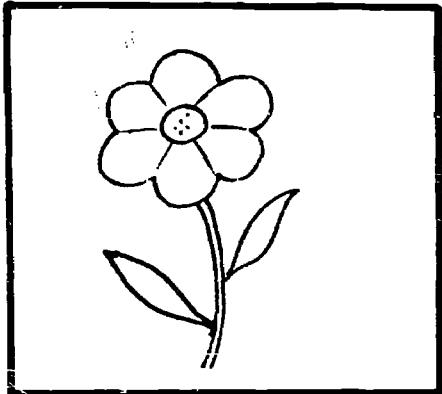
6

Paste on the picture that shows what the numeral says.

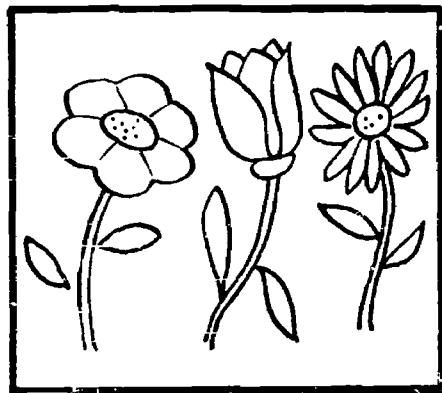


Cut out one box at a time.  
Paste it above the numeral that tells how many  
are in each box.

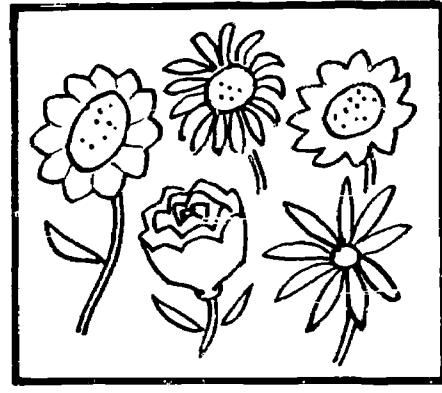




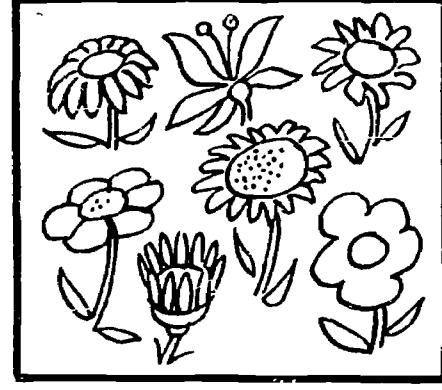
2  
1  
7  
5



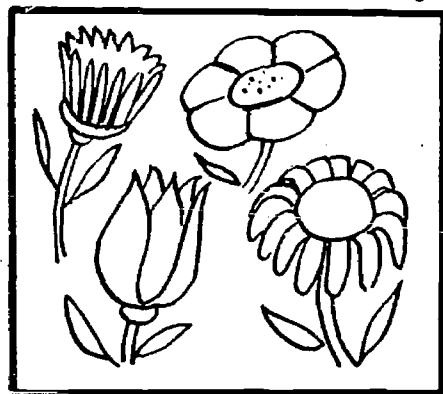
4  
3  
6  
5



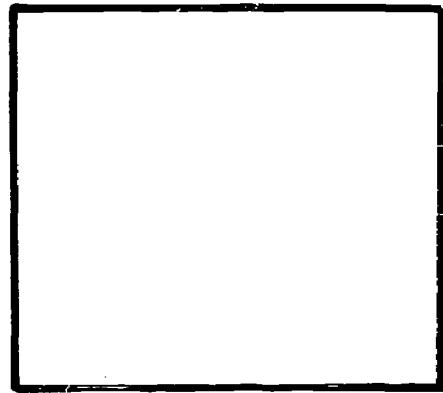
7  
2  
0  
5



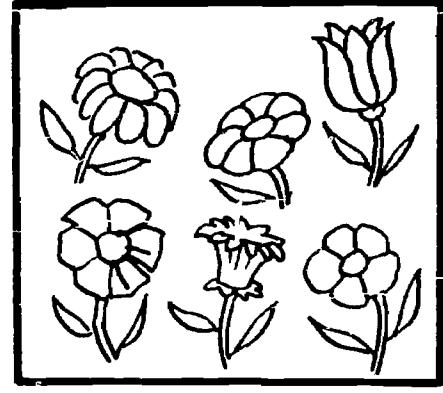
7  
6  
2  
1



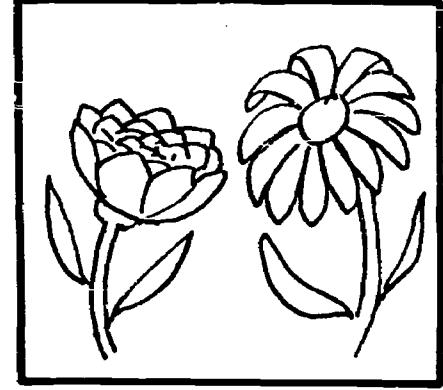
6  
4  
2  
0



7  
3  
0  
1



4  
6  
2  
3



2  
1  
4  
0

Circle the numeral that tells how many flowers are in the box.

1		4		6		8	
2		3		7		6	
3		5		8		9	
4			1	9		7	
5			2	0		0	

Make as many tally marks inside the box as the numeral  
next to it says.

Recognizing Numerals;  
Telling Time

The

BIRTHDAY PARTY



## THE BIRTHDAY PARTY

One day Linda came home from school all excited.

"Mother, Mother," she called. "Dick is having a birthday party.

May I go? May I, Mother? Please?"



## THE BIRTHDAY PARTY

"When is his party?" asked Mother.

Linda gave Mother the card. It said,

Please come to my birthday party

TIME: Two o'clock

DATE: Saturday

WHERE: 4610 - Columbus Avenue

Dick Green

"Yes, dear. You may go," said Mother.



## THE BIRTHDAY PARTY

"There is one thing, Linda," said Mother. "I have to go away this Saturday to work for the Mother's Club, so you will have to get ready all by yourself."

"I can do it," said Linda.



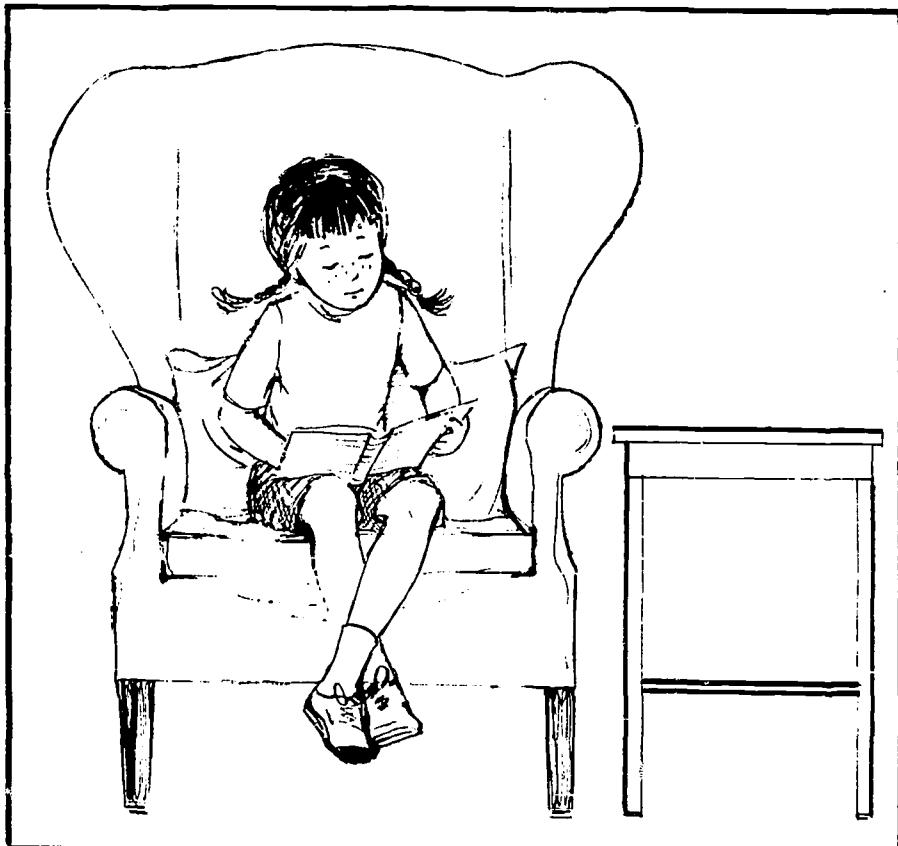
"Oh, Lucky," she said to her dog. "I'm going to a birthday party!"

## THE BIRTHDAY PARTY

At last Saturday came. When Linda was eating breakfast, Mother and she talked about what she would wear to the party.

Before Mother left, she asked, "Linda, are you sure you know how to tell when it is two o'clock?"

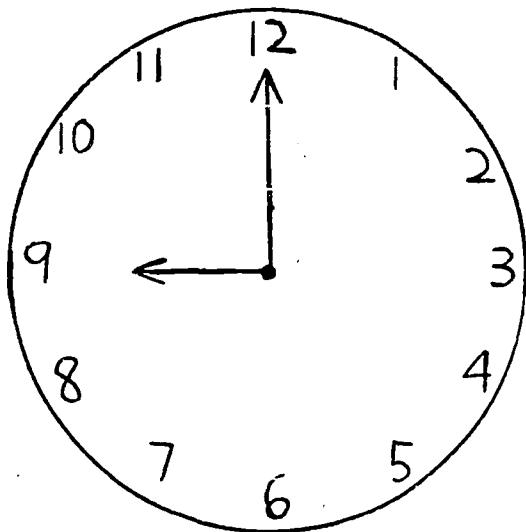
"Yes, Mother," she said. "I know."



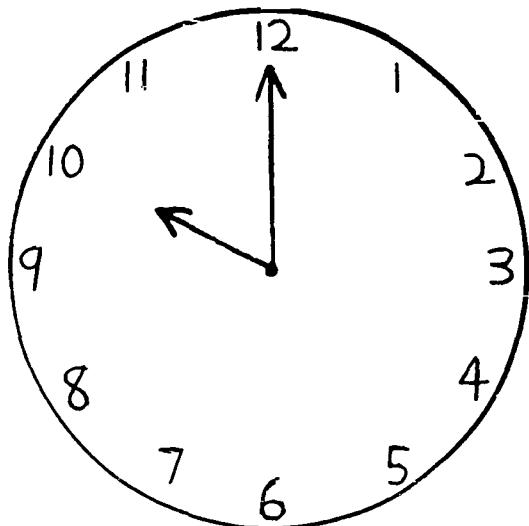
After Mother left, Linda played with her toys. She read her library book. My, time went so slowly.

## THE BIRTHDAY PARTY

First the clock said -



Then it said -

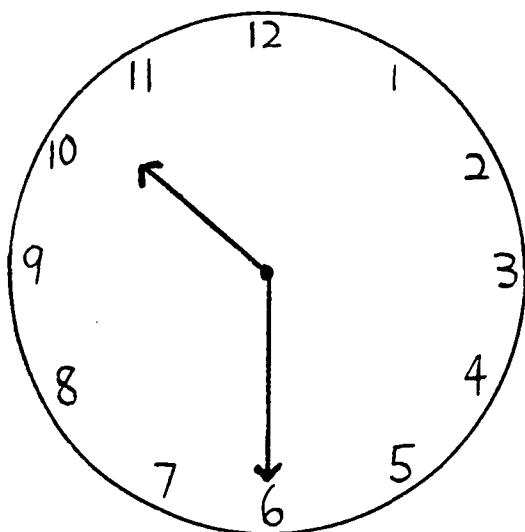


Linda thought she should get ready so she would be on time for the party.

## THE BIRTHDAY PARTY

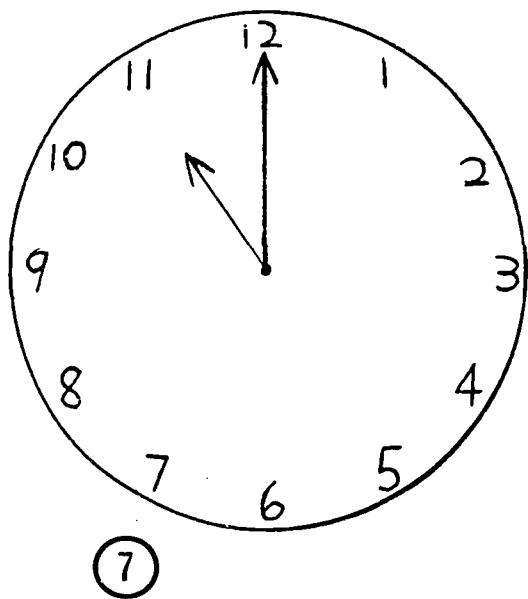
She washed herself so she would be nice and clean.

Then the clock said -



Linda put on her pretty party dress.

The clock said -



## THE BIRTHDAY PARTY

Then she combed her hair.

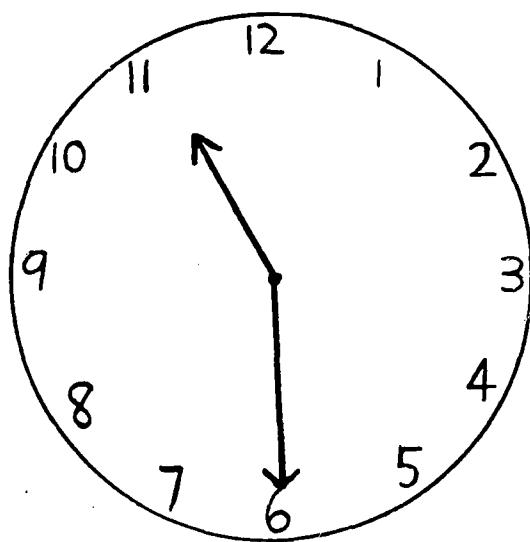


She took a long time because she wanted to look pretty at Dick's party.

She found just the right color ribbon to put in her hair, too.

## THE BIRTHDAY PARTY

Then the clock said -



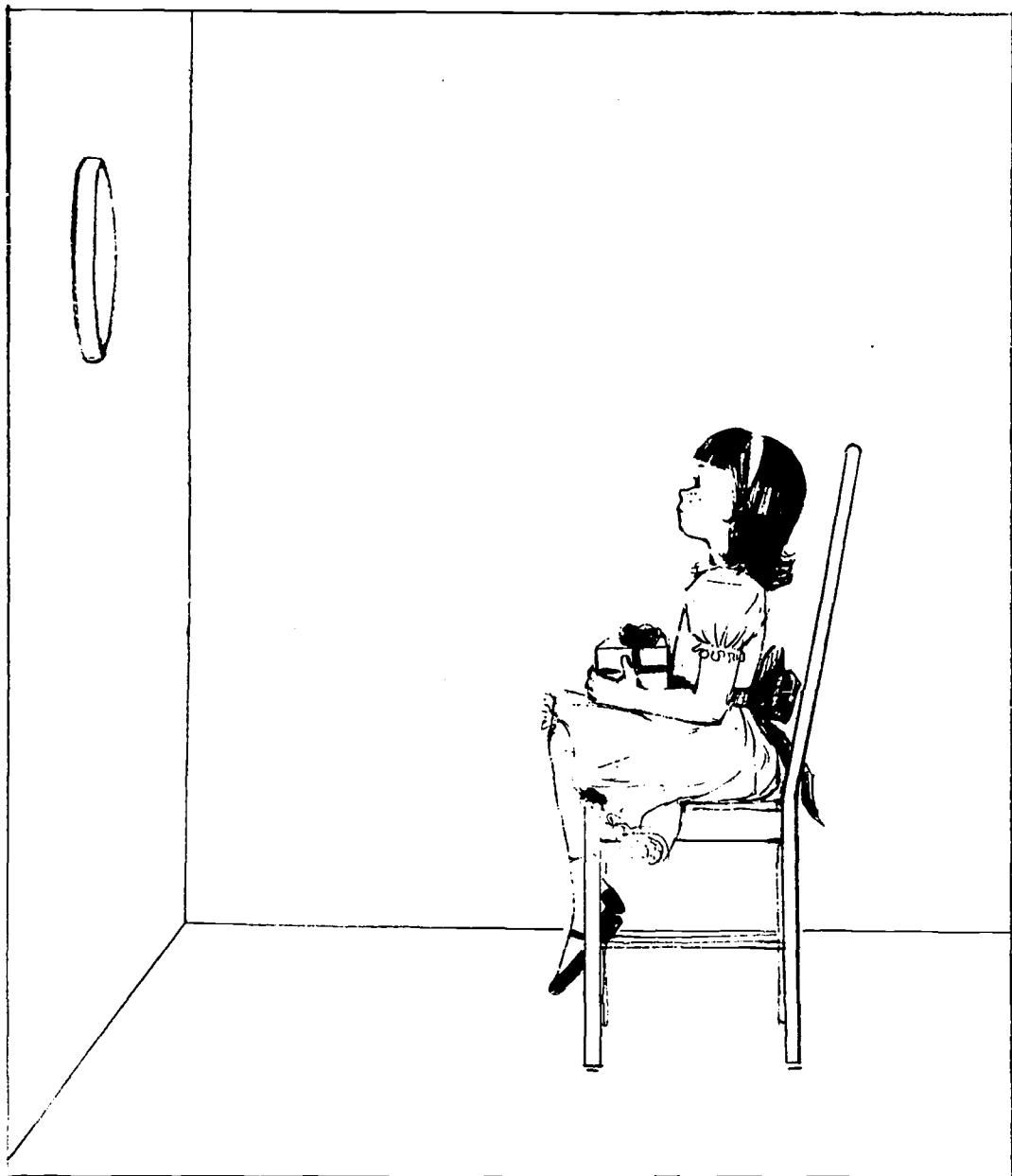
Linda went to find the birthday present she and Mother had for Dick.

Linda knew Dick would like the present.

## THE BIRTHDAY PARTY

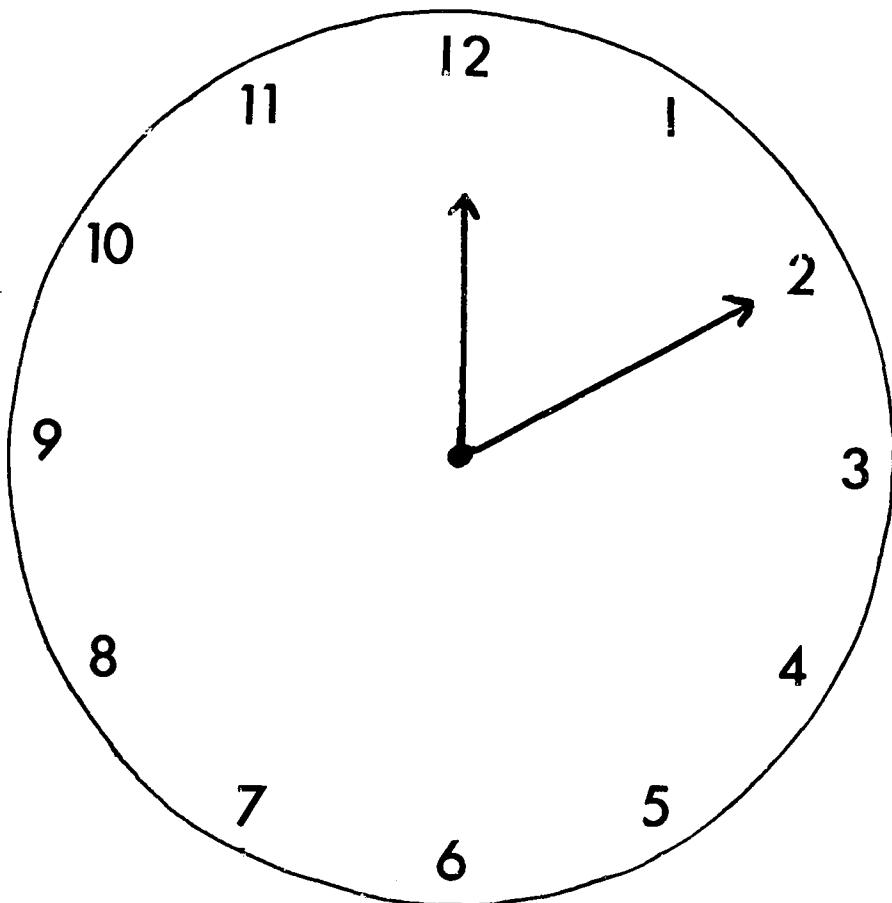
Linda sat down in front of the clock and just waited.

She waited and waited and waited.



## THE BIRTHDAY PARTY

Then the clock said -



"There." said Linda. "It's 2 o'clock. I can go to the party now."

She ran all the way to Dick's house and rang the bell.

## THE BIRTHDAY PARTY



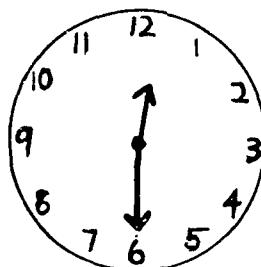
"Why, Linda," said Dick's mother. "It isn't time for the party yet. It's just a little after 12. The party is at 2 o'clock."

"Oh," said Linda, "I thought the clock said 2 o'clock."

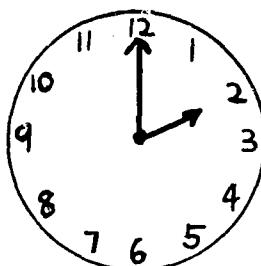
## THE BIRTHDAY PARTY

So Linda went back home to wait.

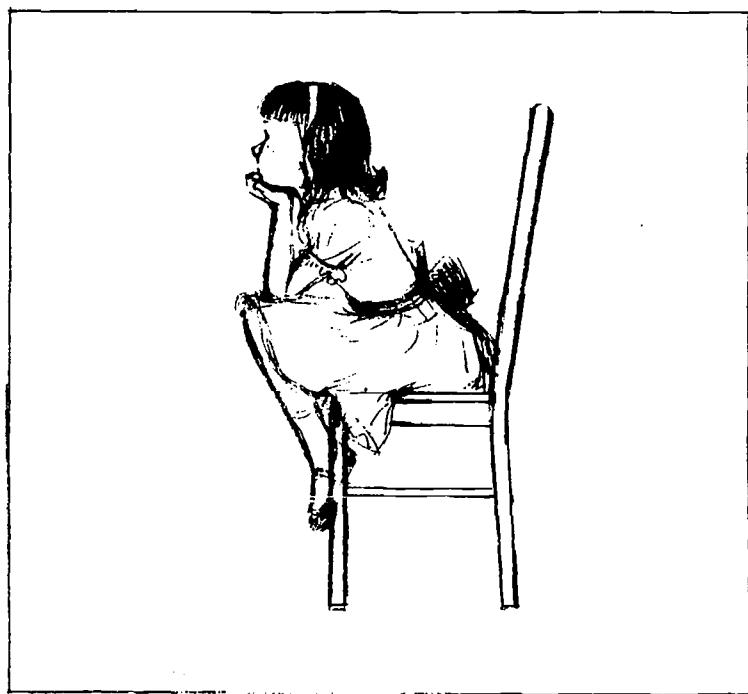
From 12:30



until 2:00



is such a long time when you are just sitting --- waiting.



## THE BIRTH DAY PARTY

Then Linda decided to play with Lucky. She threw his ball and  
Lucky ran after it.

When it was almost 2 o'clock, Linda's mother called on the telephone.



"It's time for you to go, Linda," she said. "Are you all ready?"

"Oh, yes," said Linda, "I'll go right away. Thanks for calling.

Good-bye."

"Good-bye, dear. Have a good time."

## THE BIRTHDAY PARTY

Linda hurried to Dick's house.

On the way she said to herself, "I wish I could tell time. I spent the whole day waiting for 2 o'clock to come. If I could tell time I could have played today. Instead, I just had to sit. I'm going to ask Mother to teach me to tell time so I don't make a mistake the next time! "



How about you? Can you tell time?

## Suggested Activities on Telling Time

1. Draw a clock face on chalk board. Explain that whenever the long hand is on the 12 it is o'clock. Linda went to the party at the wrong time because she didn't know that when the long hand is on 12 the short hand shows 'what' o'clock it is.
2. Have the children draw a given hour on the chalkboard clock face. Use a toy clock or large (real) clock if available and have children move the hands to a given hour.
3. Play the game, "I Went Shopping". One child may be "it" and stand in front of the class. Using a toy clock, he may turn the hands to such a time as 10:00. He should say, "This is the time I went shopping....what time was it?" The first child to raise his hand and tell the time correctly then becomes "It" and adjusts the clock to the time of his choice. The examples of time that the children use depend on their present knowledge of time telling; the game becomes more fun as more times are learned.
4. Play the game, "I Am a Clock". One child is chosen to be the "Clock". He should stand at the front of the room holding a yard stick in one hand to represent the long hand and say, "I am a clock". While indicating the time with his arms, he should say, "My short hand is on (number) and my long hand is on (number). What time do I tell? The first child to raise his hand and tell the time correctly becomes the new "Clock".
5. Play the game "Catch the Train"  
This should be played in the gymnasium or on the playground. If played in the classroom, use a smaller group of children.

Divide the class in half. Choose one child to be conductor. Have half of the class line up behind him, with their right hand on the right shoulder of the child in front, forming a train of cars. This group chugs around the room, softly imitating a train.

The other half of the class are the passengers and they wait at one end of the room. There should always be an equal number of cars and passengers, excluding the conductor. If the class cannot be divided exactly, choose two conductors instead of one.

The conductor says, "Train leaves at (time)". He then gives various times by describing hand positions of the clock. In other words, he first names the hour, then correctly describes it. Example: "3 o'clock - long hand on the 12, short hand on the 3."

When the time of departure has been correctly described, the passengers and the conductor try to "catch the train" by joining their right hands to the left hands of the cars on the train. Only one passenger to each car; the child left without a car becomes the new conductor. The children who were the "train" become the passengers and the passengers form a new train.

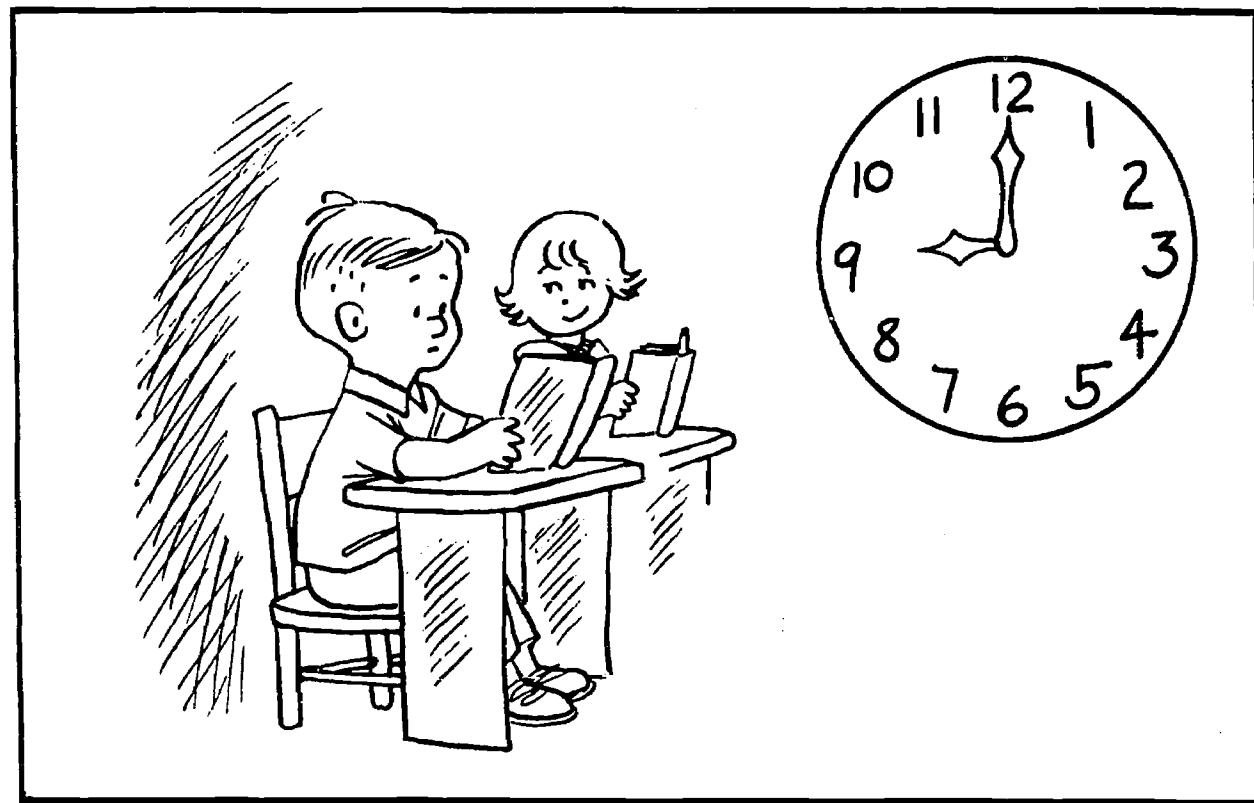
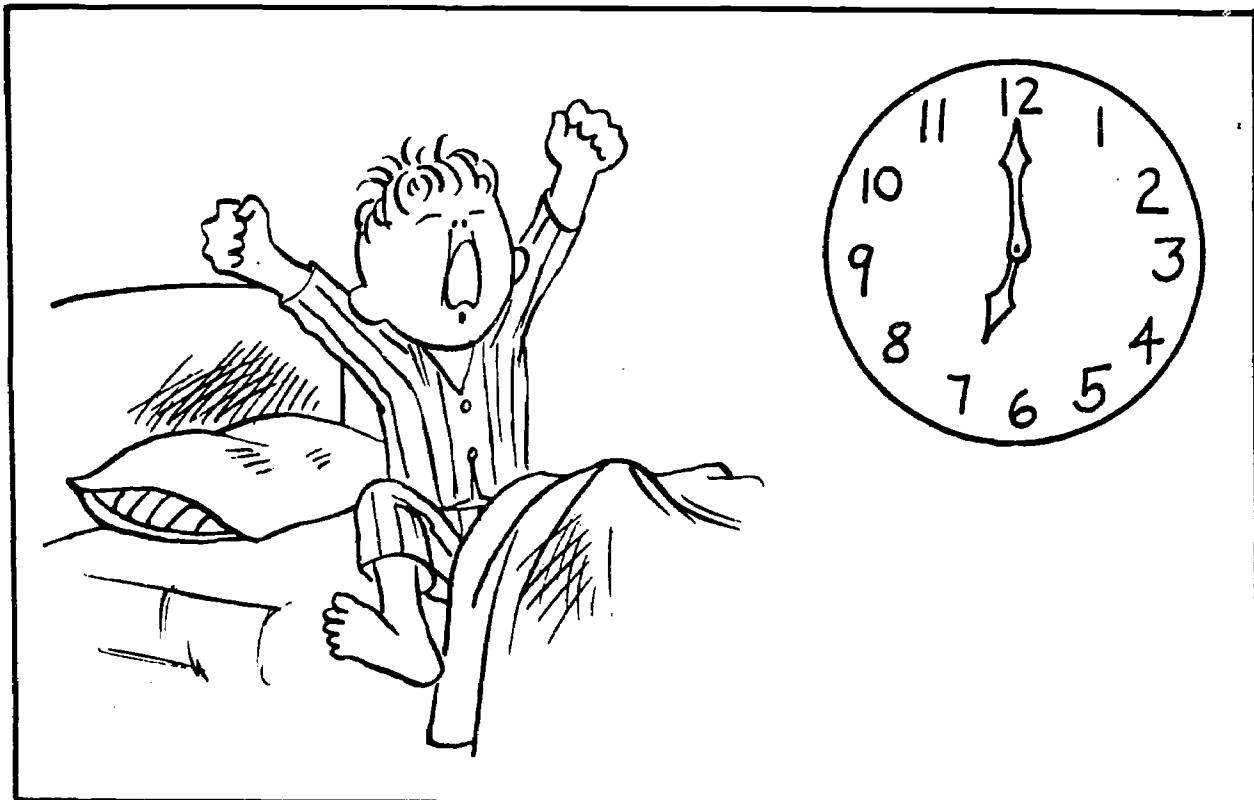
6. Call attention to the real classroom clock whenever there is an opportunity to read the time on the hour. Reinforce the idea that the long minute hand points directly at 12 on the hour and the short hour hand directly to the hour.
7. The concept of A.M. and P.M. is not introduced in this unit. The following terms may be used to differentiate between A.M. and P.M.: in the morning, in the afternoon, and in the evening. "Twelve o'clock noon" and "twelve o'clock midnight" may also be used in specifying the time.

### Commentary on Worksheets 17, 18, 19

The following worksheets are designed to give the children practice in telling time on the hour and associating specific hours with events in the day.

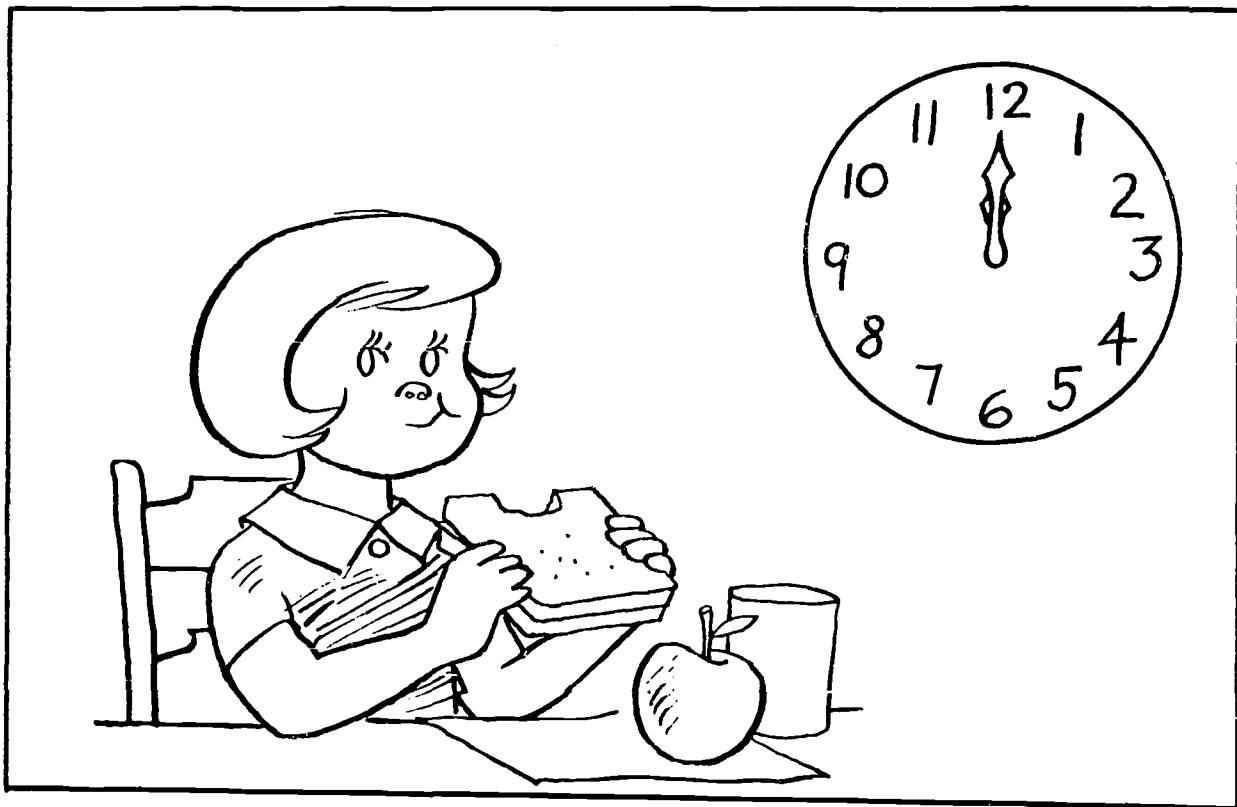
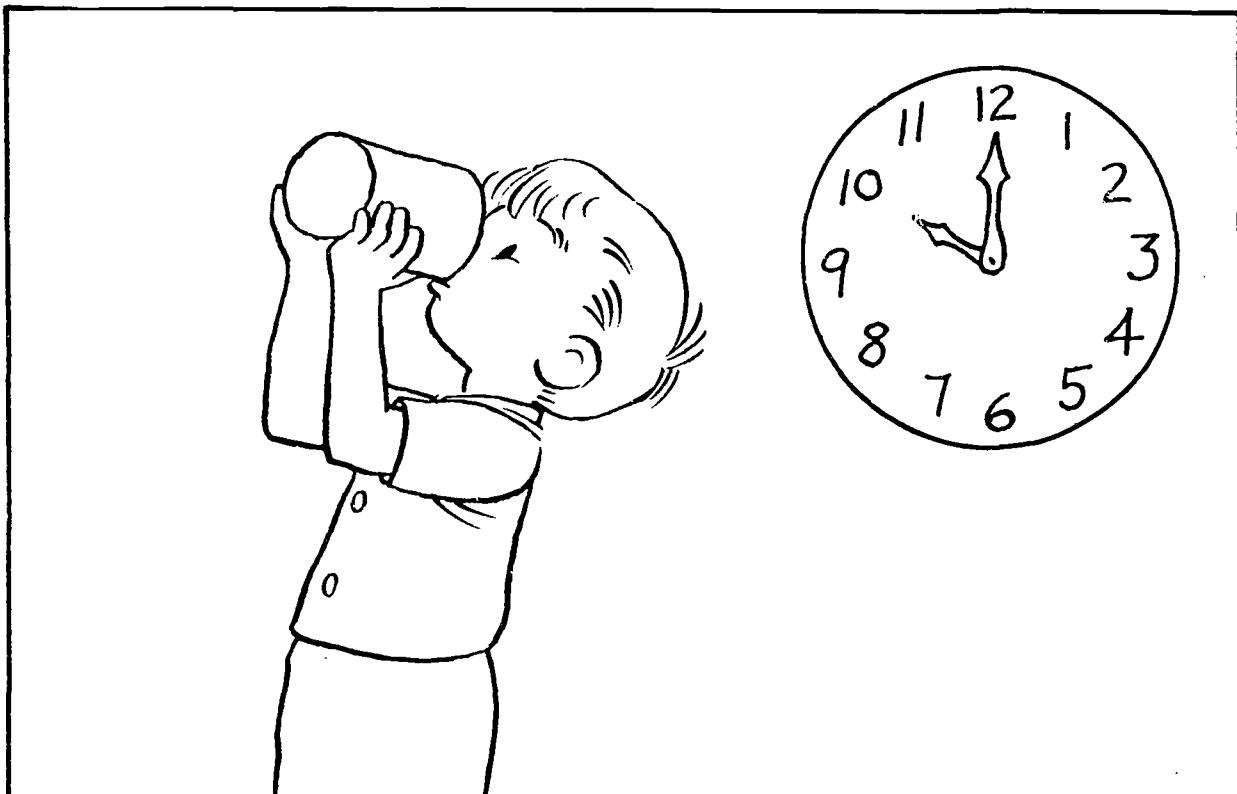
Distribute Worksheet 17. Ask the class the questions on the worksheet. Discuss the illustrations and have class members show the hours on the classroom model clock. Worksheet 18 may be presented in the same manner as Worksheet 17.

Distribute Worksheet 19. Tell the class that the minute hand is pointing to twelve. Discuss what is missing. (The hour hand) Have the children count to make sure that none of the numerals are missing on the clock face. Have them point to each numeral as the numbers are said from one through twelve. Instruct them to draw in the hour hand pointing to any hour they wish. Then they are to illustrate an activity which is appropriate to that hour. 12 by 18 inch manila paper should be provided for this part of the activity.



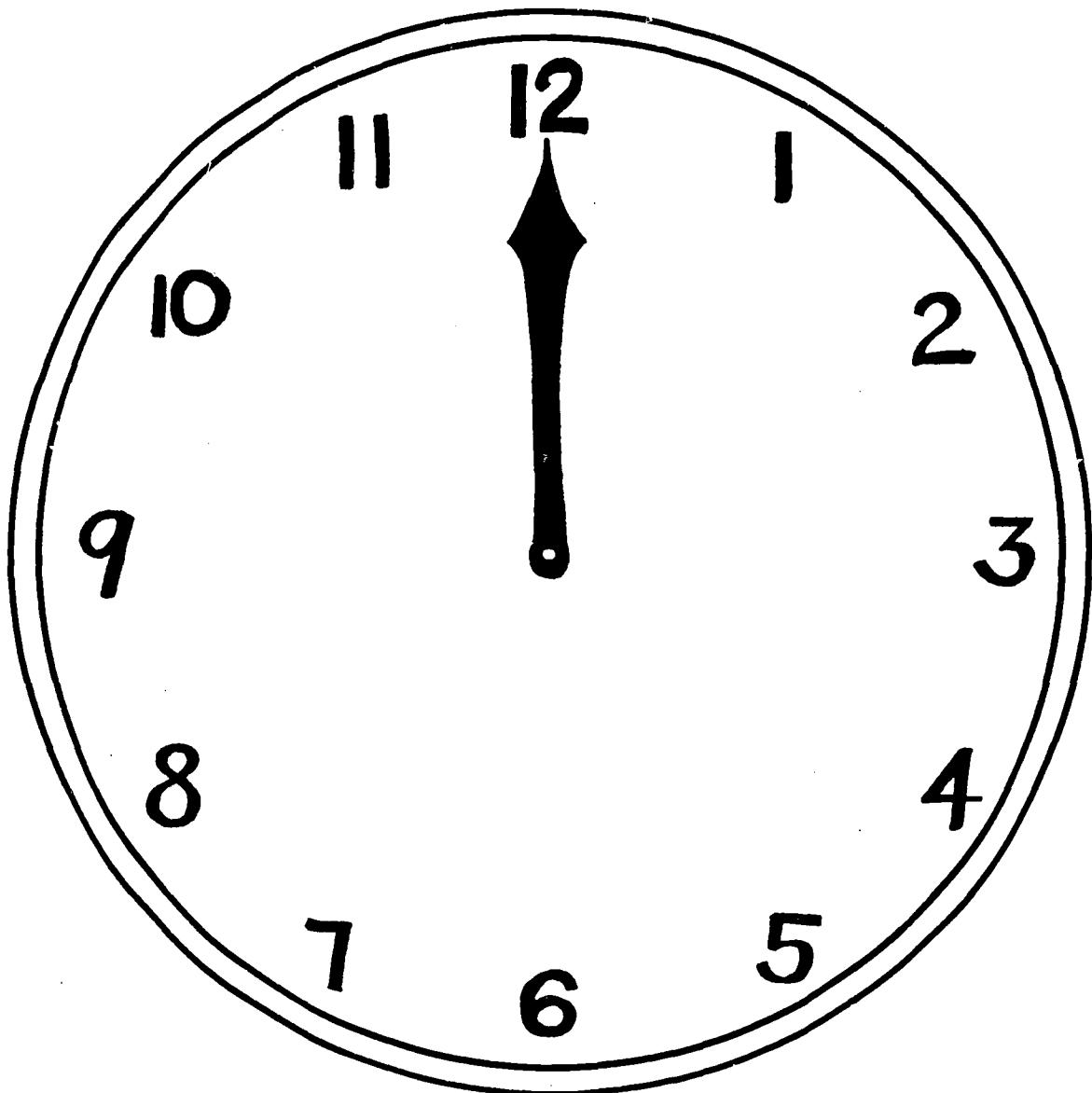
What time does Billy get up?

What time does Billy's class look at books?



What time is milk served?

What time is lunch?



Draw in the hour hand to any hour.

Make a picture showing what you do at that hour.

## Commentary on Worksheet 20

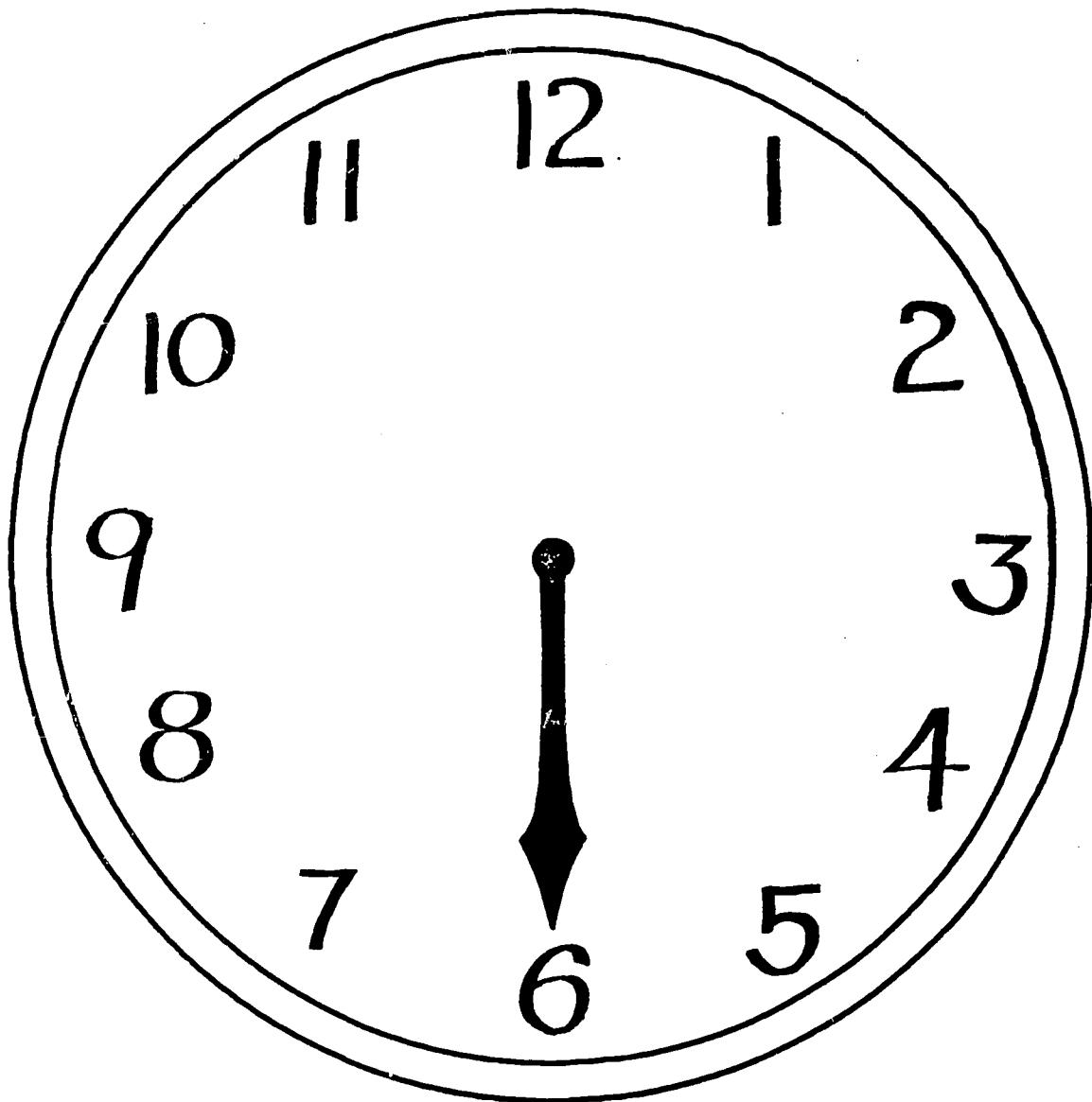
It is felt that after the children can readily tell time on the hour, they might be ready for the "half hour". It should be explained to them that whenever the big hand is on the 6 it is "half-passed".

The teacher should demonstrate the "half-passed" idea on the chalkboard, a toy clock or a large (real) clock.

Explain that like having half an apple indicates that half the apple is gone, the "half-passed" indicates half-an-hour of time passed. It is half of a whole hour. On the clock it will be noted that when the big hand is on the 6 the hour hand is halfway between the hour it is past and the next hour.

Repeat activities 2-6 on page 57 to provide opportunities for telling time on the half-hour.

For Worksheet 20 have the children draw in the hour hand half passed any hour. Emphasize that the hour hand is half way between the hour it is passed and the next hour. Check each paper to see that this is understood when each child tells the time on his clock.



Draw in the hour hand to show a time half past an hour.  
What time have you drawn on your clock?

## Commentary for Worksheets 21-25

The following sequence of activities may be used to introduce the class to the writing of numerals. It is left to the discretion of the individual classroom teacher as to whether or not their particular class has the readiness for writing.

Distribute Worksheet 21. Read the rhyme about numeral one: One is fun, Straight down and you're done. Demonstrate the correct printing of numeral one on the chalkboard. Have the children trace the numeral one in the first box. Explain that the little x shows where to begin printing the numeral. Have the children trace the dotted lines in each box, first with their pointing fingers, then with their printing pencils or crayons. The bottom row provides an opportunity for the children to print the numerals independently.

Follow the same procedure for the other numerals in Worksheets 21-25.

One is fun  
Straight down and you're done.

Curve around the trace  
Then come back  
The train says, "Two, two, two."

See me make a three  
Curve around and curve around  
It's simple as can be.

Down and over  
Then down some more  
When you finish you've made a four.

To make a five go down and around  
Put a line across the top  
And see what you've found.

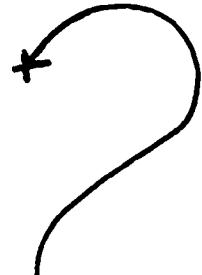
Curve down and make a loop  
The numeral six rolls a hoop.

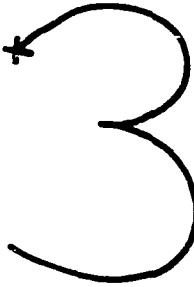
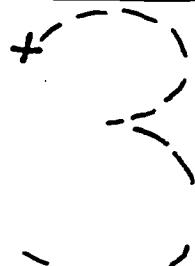
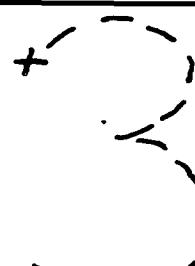
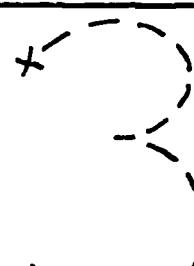
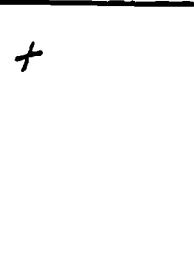
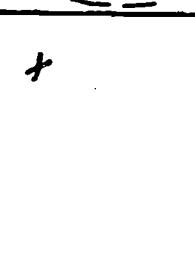
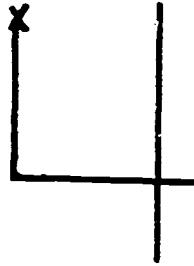
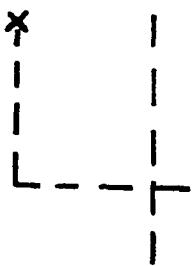
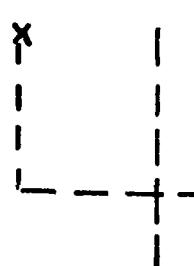
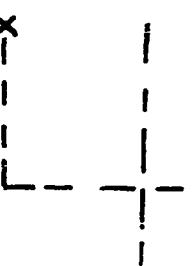
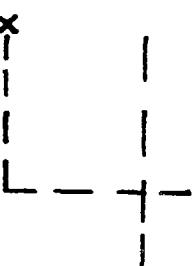
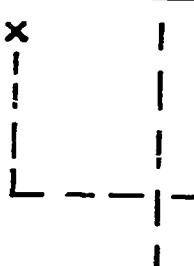
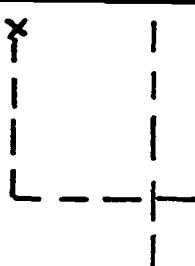
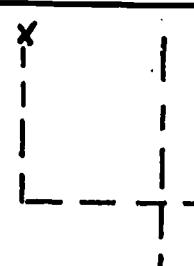
To make a seven go across the top  
Then drop down and come to a stop.

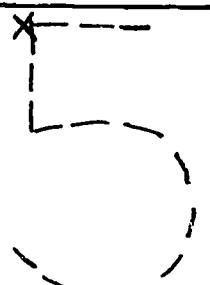
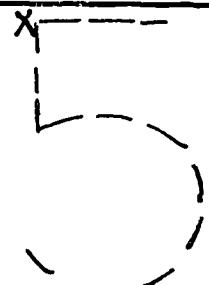
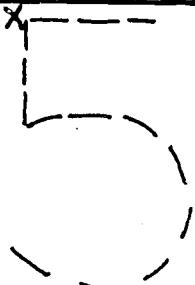
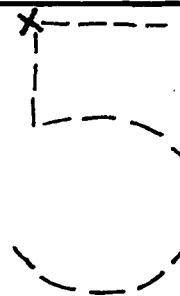
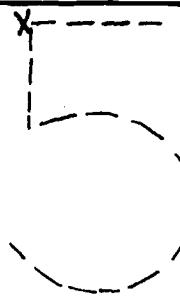
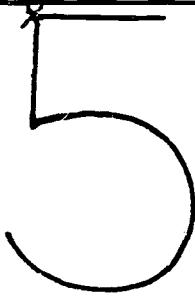
Make an S but do not wait  
Go right back up to make an eight.

Make a circle, then a line  
That's the way to make a nine.

Make a 1 and a zero next to it  
To make a ten, that's how you do it.

	x	x	x	x
x	x	x	x	x
x	x	x	x	x
				
+ —	+ —	+ —	+ —	+ —
x	x	x	x	x



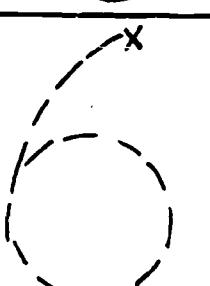
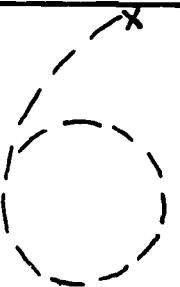
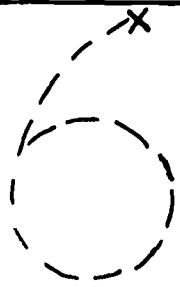
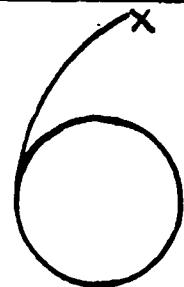
x

x

x

x

x



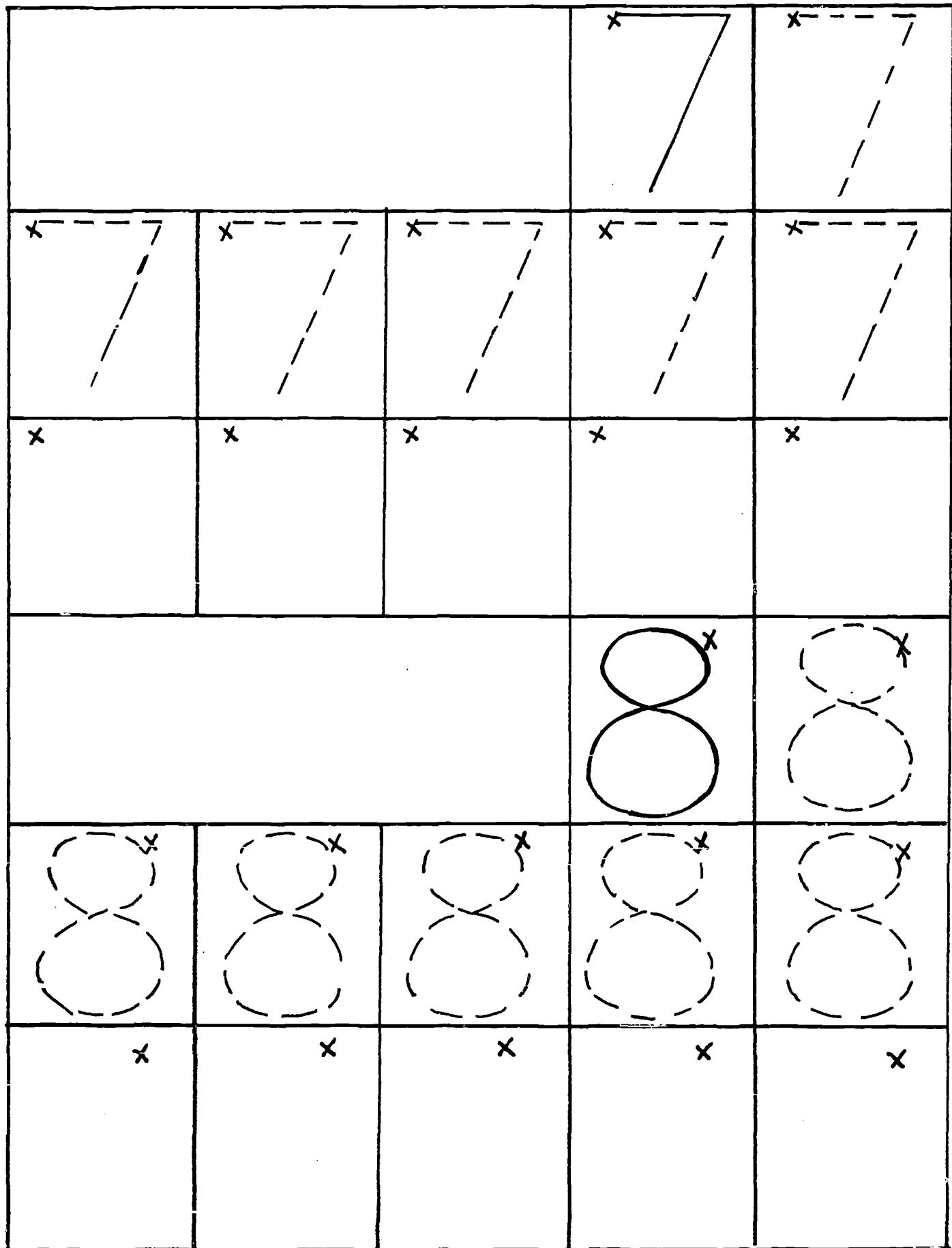
x

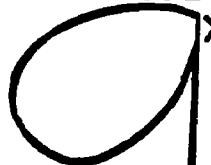
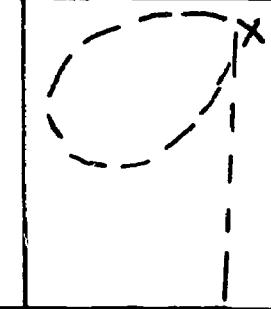
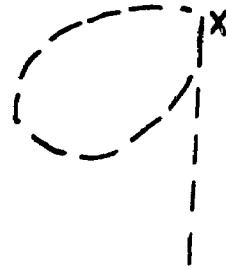
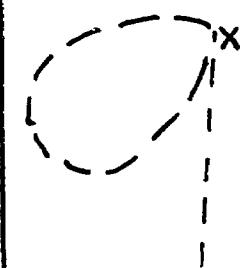
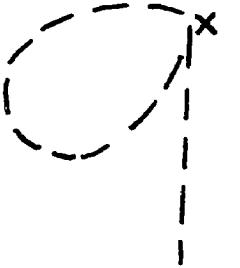
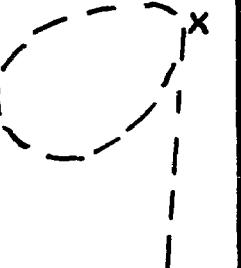
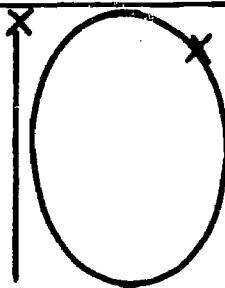
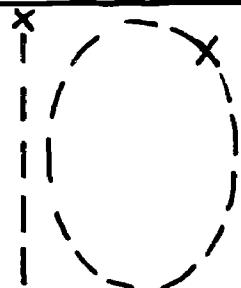
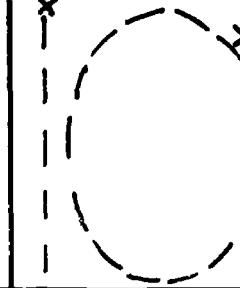
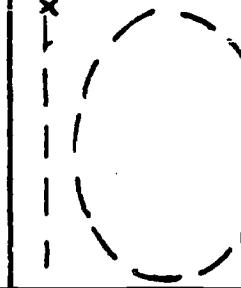
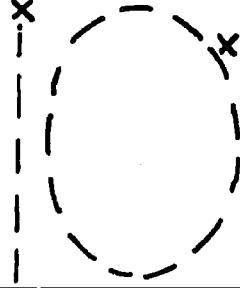
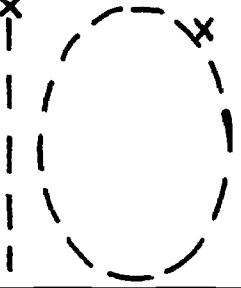
x

x

x

x



				
				
x	x	x	x	x
				
				
x x	x x	x x	x x	x x

## Suggested Activities on Writing Numerals

1. Provide opportunities for the children to write the numerals at the chalkboard relating to their background of experiences. For example: print the numeral that tells how many wheels are on a tricycle, how many eyes we have, how many toes we have, how many buttons are on John's jacket, etc.

### 2. Who Can Write My Numbers

Print an important telephone number such as the police department number on the chalkboard. The child who is "It" prints each numeral on the board in the proper sequence from left to right as it is read by the class. "It" calls on another child to read the number that he has written. If the number is written correctly, the child he chose becomes the new "It". If the number is not correct the child he chose tries to correct it.

Each child might have an opportunity to write his own telephone number on the chalkboard. The teacher should print the station letters if they are a part of the telephone number. Example: FE 5 - 3461.

### 3. Missing Numerals

Use the chalkboard for this activity.

Start with a series of three numerals. Leave a blank for the missing numeral. If the child fills in the blank correctly, he may write the next set of numerals and blank.

The first series of numerals may deal with two numerals and then the blank. For example:

1 2 \_\_\_\_\_

The second set may deal with two numerals and the blank in the middle:

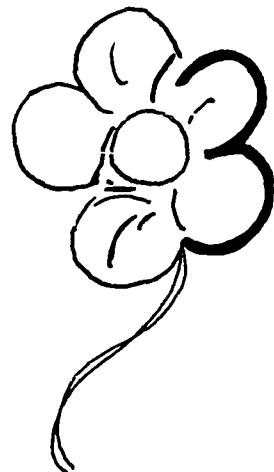
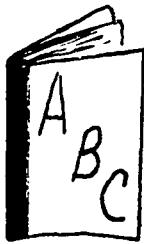
3        5

The third set may deal with the blank coming first and then the two numerals:

         4    5

#### 4. Art Activity with Numerals

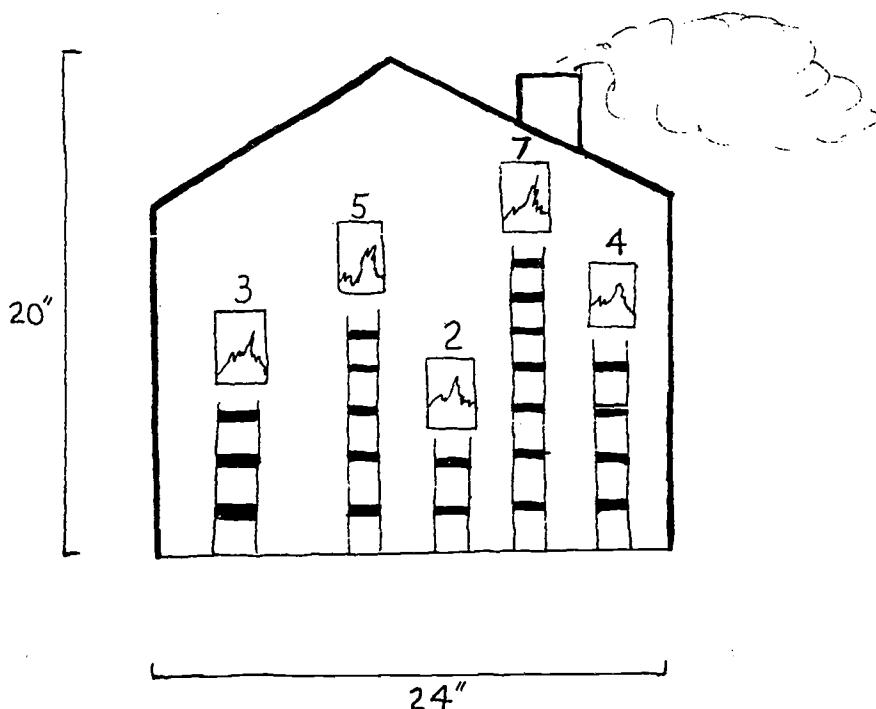
Distribute 12" by 18" manila paper. The children may print any numeral they wish in large size with their crayons. They then draw a picture with the numeral incorporated into the design. For example:



5. Draw a large outline (about 20" by 24") on the chalkboard. Draw in several windows at different levels from the bottom line of the house. Tell the children that there is a fire in the house and they can rescue the people in the house if they can climb the ladders and print the numeral telling how many steps they've

climbed to reach the windows. Draw in one ladder at a time.

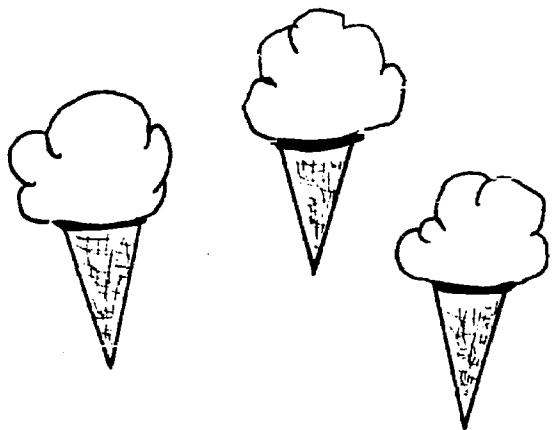
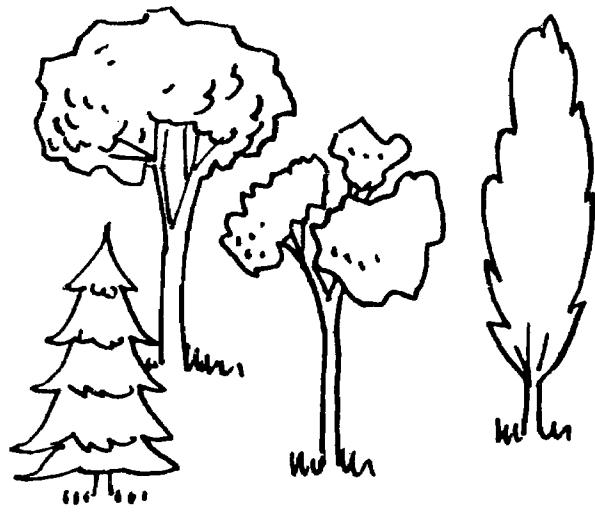
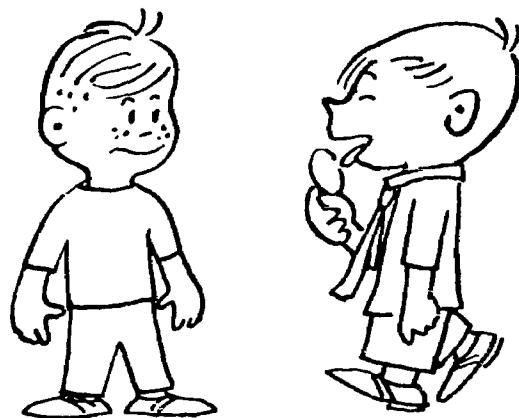
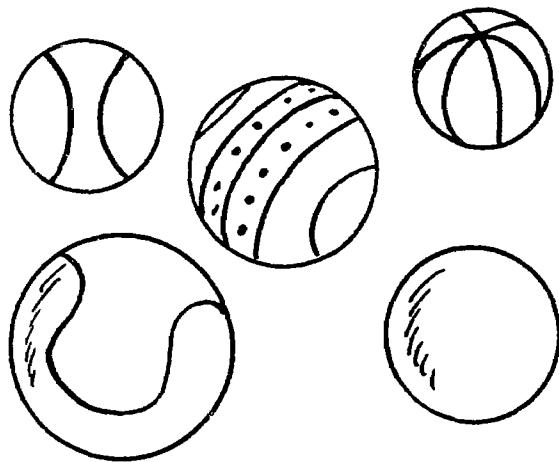
See illustration.



#### Commentary on Worksheets 26-30

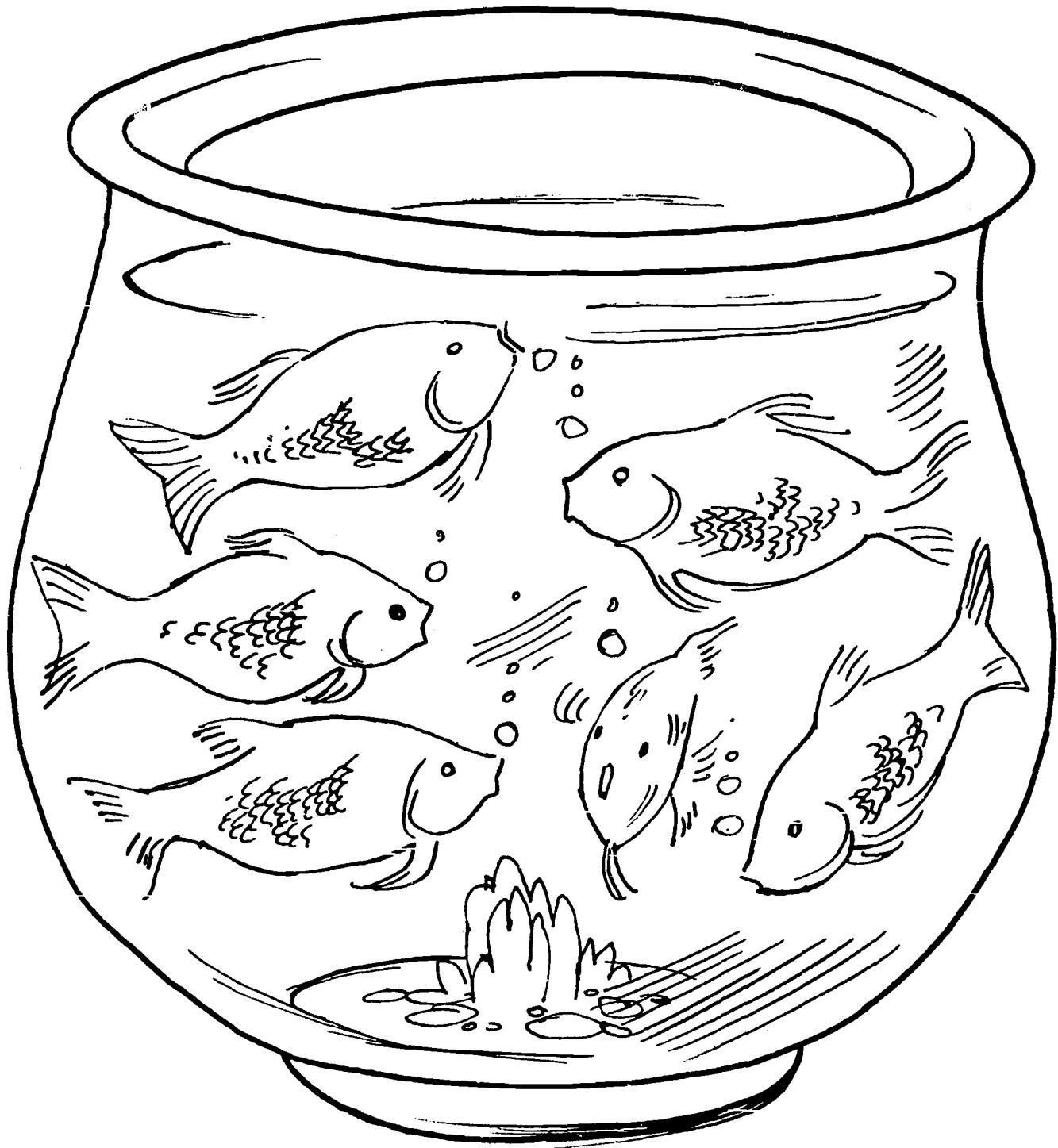
Worksheets 26-28 provide counting and printing practice. Read the directions on the worksheets to the class. The directions are self-explanatory. The numerals at the bottom of each worksheet are for purposes of reference.

Worksheets 29 and 30 involve filling in the missing numeral.



In each box print the numeral that tells how many members are in the set.

0 1 2 3 4 5

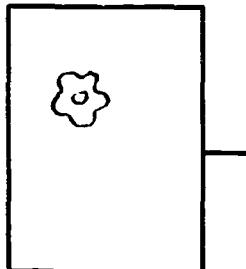


How many fish do you see?

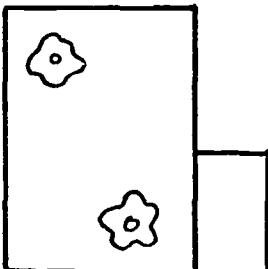
Print a numeral on each fish starting with the numeral one and going in counting order.

1 2 3 4 5 6

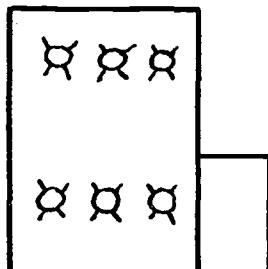
A



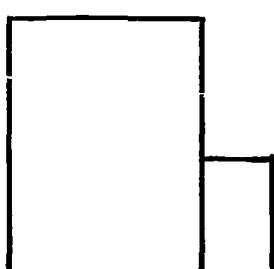
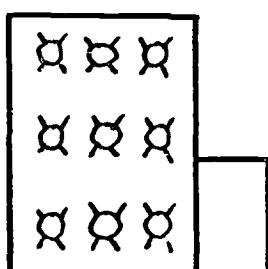
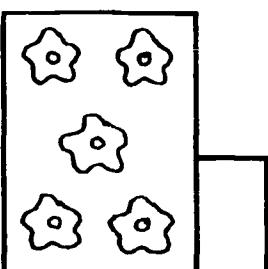
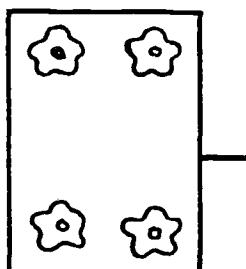
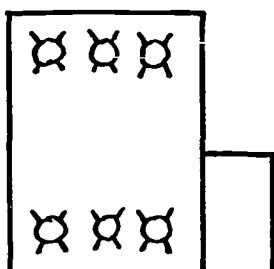
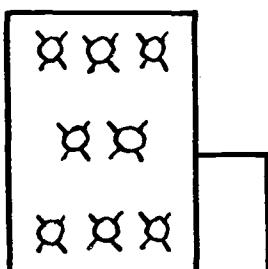
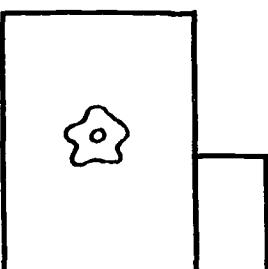
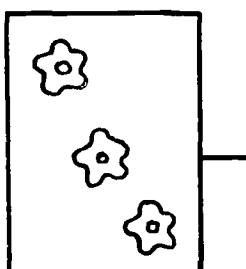
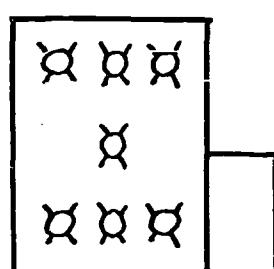
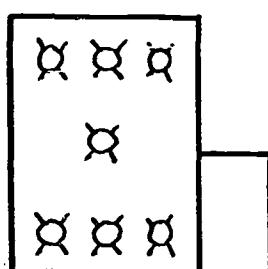
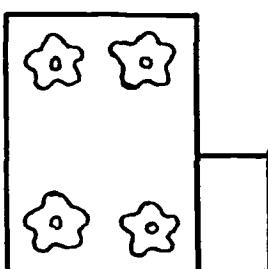
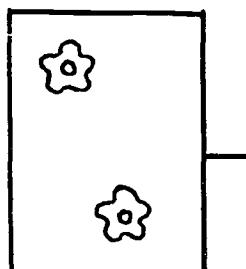
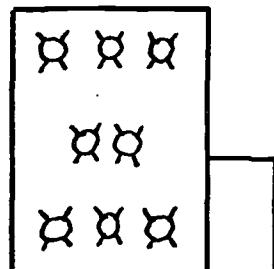
B



C



D

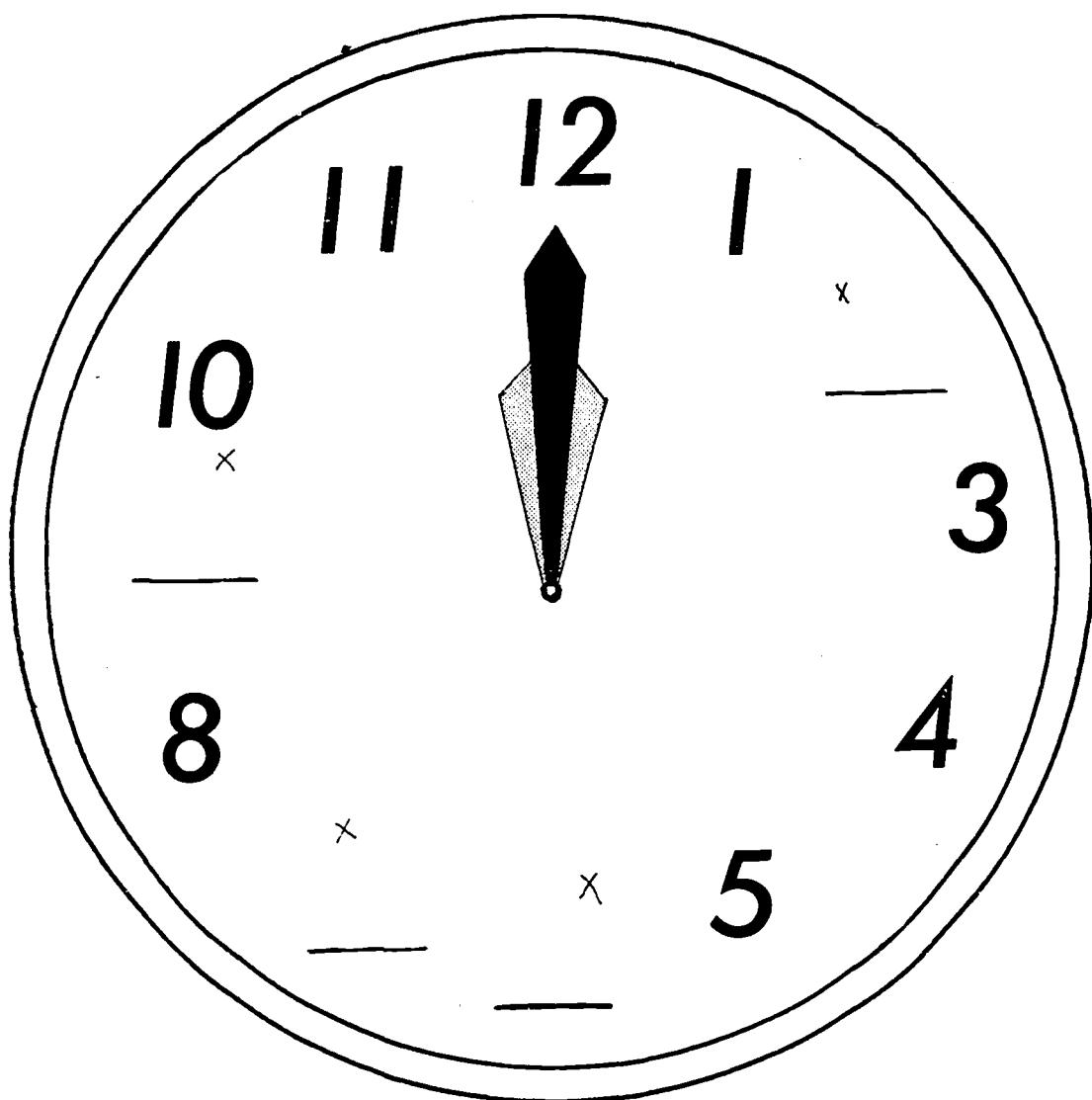


In the box to the right of each set, print the numeral that tells how many members are in the set.

1 2 3 4 5 6 7 8 9 10

Fill in the missing numerals.

1	2	x	4	x	6	7	x	9	x	x
		—		—			—		—	



Fill in the missing numerals.

XXXX  
XXX

XXXX  
XXX

XXXX  
XXX

XXXX  
XXX

7

10

4, 5, —

6, —, 8

—, 2, 3

6, 7, —

1, —, 3

—, 6, 7

2, 3, —

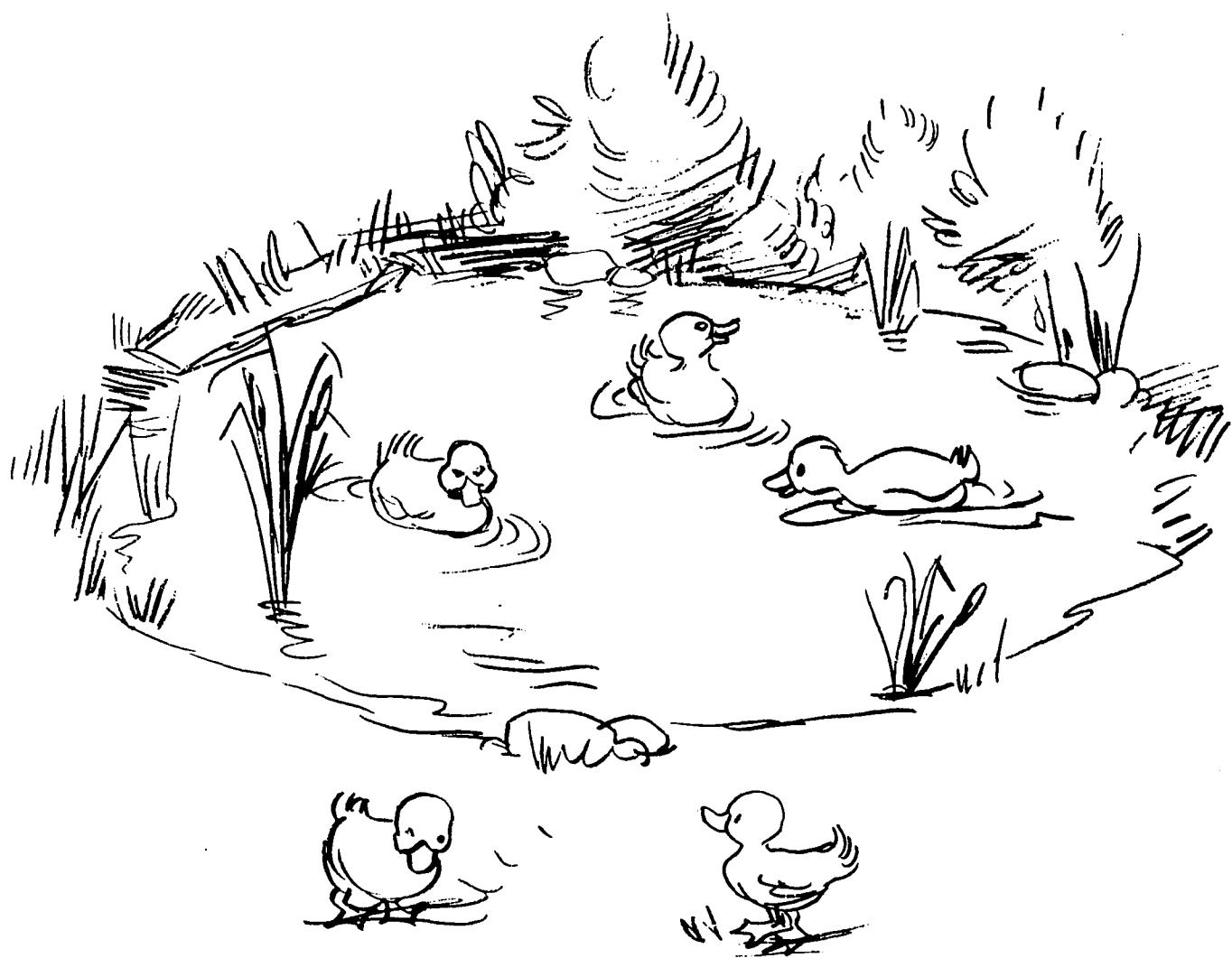
8, —, 10

—, 4, 5

Commentary on Worksheet **31**

This worksheet might precede or follow a few similar activities.

Do not bring out the concept of addition at this time. In the Minnemast program addition is formally introduced with the number line.



How many ducks are in the water? \_\_\_\_\_

How many ducks are outside the pond? \_\_\_\_\_

How many ducks can you see altogether? \_\_\_\_\_

**Note:**

The material of the remainder of this unit, on union and intersection, is repeated in Unit VII. Classes which will use Units VII and IV during the same school year should skip this material at this time, proceeding directly to Unit V. Classes not using Units IV and VII during the same school year should try this material at this time.

## Teacher Background on Union and Intersection

For any two given sets, say A and B, there are determined two important associated sets. These two sets are called the "union of A and B" and the "intersection of A and B". The underlining on "the" is to emphasize that each of these sets is unique for each pair of sets, A and B.

### Union

The union of two sets, A and B is the set in which each member of A and each member of B is a member of the union and conversely, each member of the union is a member of A or B or of both A and B.

Since the English language lacks precision, symbols are often used in mathematics. The symbol to denote union is **U**. The union of A and B is written A **U** B.

Examples:

Consider a particular 2nd grade class and let A be the name of the set of boys in that class and B the set of girls. A **U** B is then a name for the set of children in that classroom. Furthermore, it is a name constructed from the names of the constituent sets.

Suppose now that in that same classroom there are children with the following names: Ellen, Eugene, Earl, Exeter, Joan, William, John and Levi. We assume that each of these names belongs to a different child. We now name some sets of children. Let

$$C = \{\text{Ellen, Earl, John}\}$$

$$D = \{\text{Eugene, Levi}\}$$

$$\text{then } C \mathbf{U} D = \{\text{Ellen, Earl, John, Eugene, Levi}\}$$

Here we have the union of disjoint sets. That is, no member of C is also a member of D.

Suppose, however, that  $E = \{\text{Levi, John, Earl}\}$

$$\text{then } C \mathbf{U} E = \{\text{Ellen, Earl, John, Levi}\}$$

$$\text{and } E \mathbf{U} D = \{\text{Eugene, Levi, John, Earl}\}$$

In the latter two instances, we have the union of non-disjoint sets. That is, Earl and John are members of both set C and of E. Also, Levi

is a member of set D and of E.

Note that the "names" are being used to describe the sets and to identify the members of the set. The "names" themselves are not members of the set.

It would be correct to write

$$E \cup D = \{Eugene, Levi, Levi, John, Earl\}$$

but we assume there is only one person named Levi. We usually avoid duplications in listing the names of the members.

### Intersection

The intersection of two sets, A and B, is the set in which each member of the intersection is a member of both A and B. The mathematical symbol for intersection is  $\cap$ . The intersection of A and B is written:

$$A \cap B$$

Two sets always have an intersection. The question is only what set is the intersection. The intersection may be empty, or in other words, the empty set. It is incorrect to say that the empty set is "false" or "imaginary" or "untrue". It is always there. You may not see it, hear it, or smell it, but it's there.

Referring back to the sets in the original examples, it is seen that

$$A \cap B = \{ \}$$

where  $\{ \}$  is a mathematical symbol used to denote the empty set.

Other intersections from the examples on the preceding page are:

$$C \cap D = \{ \}$$

$$C \cap E = \{Earl, John\}$$

$$D \cap E = \{Levi\}$$

## Suggested Activities on Union and Intersection

1. Ask the boys wearing tennis shoes to line up on one side of the room.  
Ask the boys wearing blue shirts to line up on the other side.

At this point it will be discovered that some boys should be in both lines, the tennis shoe line and the blue shirt line! This presents a problem to be solved - how to arrange the two sets so that it is easy to see who is in which set. Have the children solve the problem.

"Which boys belong to both the set of boys with tennis shoes and the set of boys with blue shirts?"

"Can you think of a way to arrange these boys to make it easier to see that they belong to both the set of boys with tennis shoes, and the set of boys with blue shirts?"

Tell the children that to show that some boys are in both sets we use a new word --★ Intersection. The boys in both sets are the intersection of the two sets. Have the boys who are in the intersection raise their hands.

"Here is another way to show the intersection of these sets. Draw a circle on the floor; the boys with tennis shoes stand within this circle. Draw a second circle overlapping the first and ask, "Which boys should stand in the second circle? Which boys belong in the overlapping area?" For additional experiences use squares, rectangles, ellipses, and irregular closed curves.

Bring out that the intersection is a subset of each of the old sets.

2. Say, "Now we are going to unite sets to make new sets. All girls with a plaid dress (or a specific color or any other means of identification) stand over here on this side of the room."

"All boys with green (or any other specific color) shirts line up on the other side."

"Now we have two sets - a set of girls with plaid dresses and a set of boys with green shirts."

"Will the members of both sets form a circle in this part of the room?"

"Now we have made a ~~★~~union of sets and our new set is a set of girls with plaid dresses and boys with green shirts."

Ask the question - "Why is (Mary) in the union of sets?"

"Why is John in the union of sets?" "Why isn't Ruth a member of the union, etc.

Ask all the members of the union to raise their hands.

Ask, "Who is a member of the union of the two sets?" (Anyone who is a member of either of the two sets or both. Anyone who is a member of at least one of the two sets.)

Bring out that each of these sets is a subset of their union.

3. Give each child with blue trousers an object; give each child wearing brown shoes another type of object; those with both objects are in the intersection - all children with one or both objects form the union of the two sets.

Have the children in the union raise one hand. Have the children in the intersection raise both hands.

4. Form union of other sets; children, toys, classroom objects, etc.

5. Place 10 objects or counters on a table. Have a child count them. Then separate the set of objects into two subsets (the intersection of these subsets should be empty).

Ask the children if they think there are just as many objects on the table now as there were before they were separated. In order to check for sure, choose another child to count the whole set again, leaving the subsets separated.

Next have a child count the counters in each subset. These numbers could be shown by a numeral card or written on the chalkboard.

Repeat this activity using a different number of objects in the group each time.

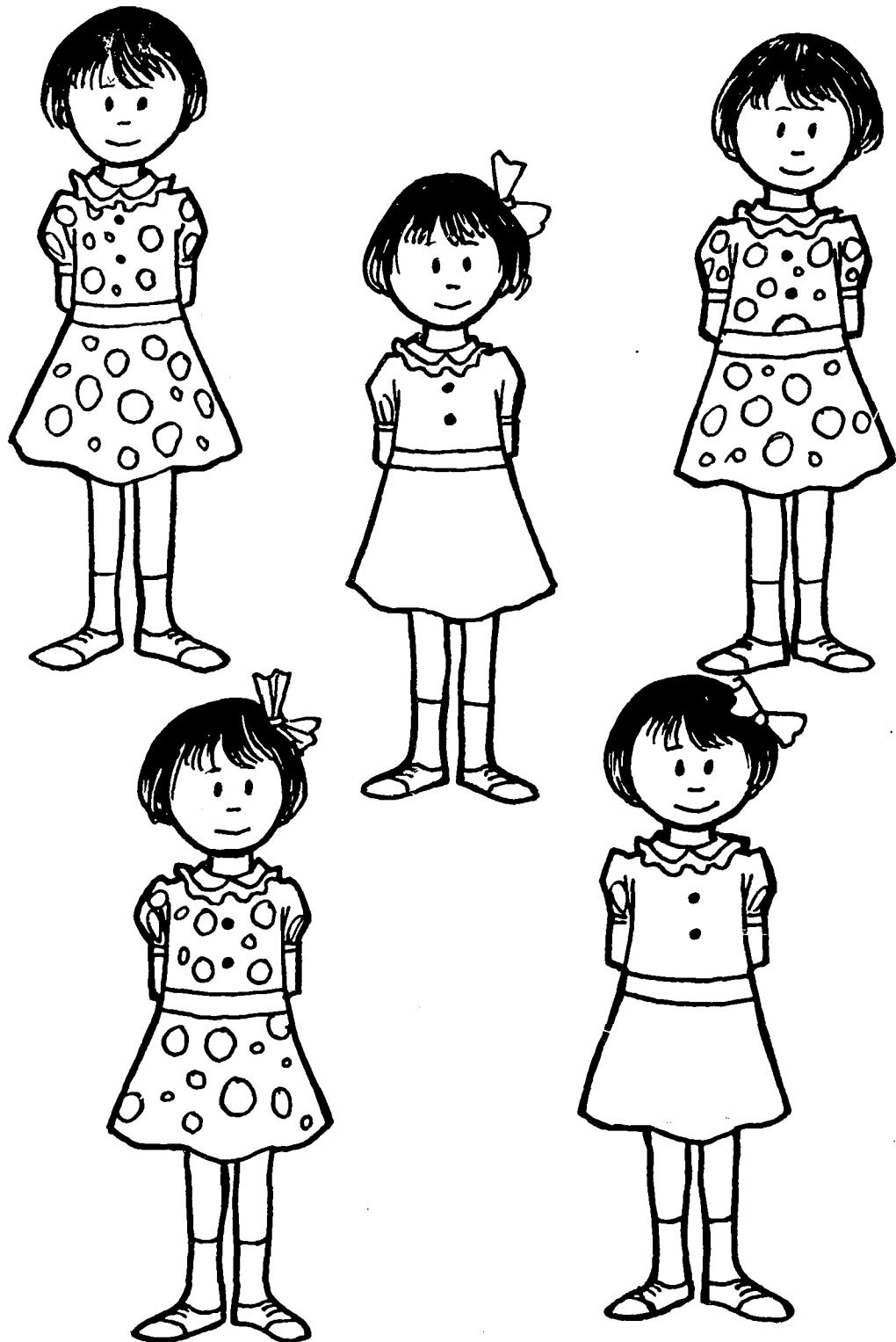
**Variation:** After separating the set into subsets, count the subsets before counting the union of the subsets.

### Commentary on Worksheet 32

Discuss the sets of girls on the worksheet: those with dotted dresses, those with plain dresses, those with hair ribbons. Direct the children to draw a closed curve around the girl who is in the intersection of the set of girls in dotted dresses and the set of girls with hair ribbons on.

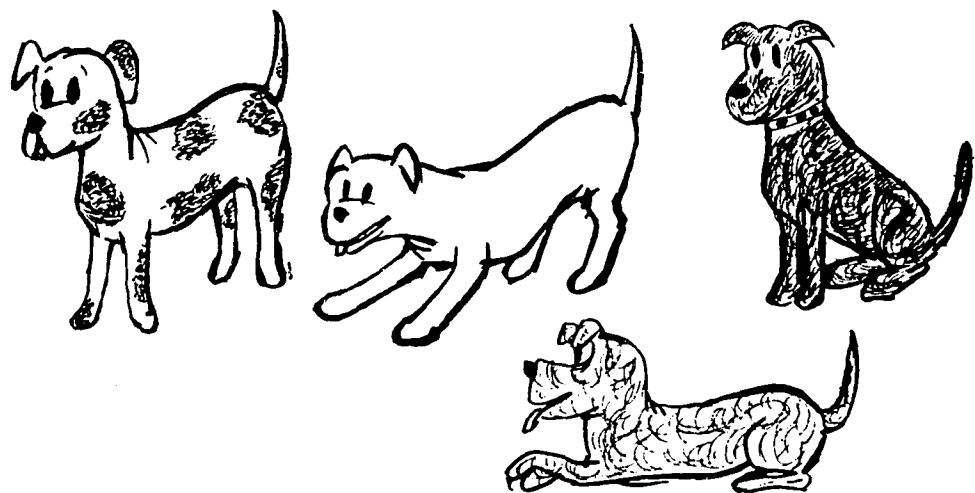
### Commentary on Worksheet 33

This worksheet should be preceded by activities of a similar nature. That is, have the children find the number of each of two sets, then ask them to find the number in the union of the sets.



Draw a closed curve around the girl who is in the intersection of the set of girls in dotted dresses and the set of girls with hair ribbons on.

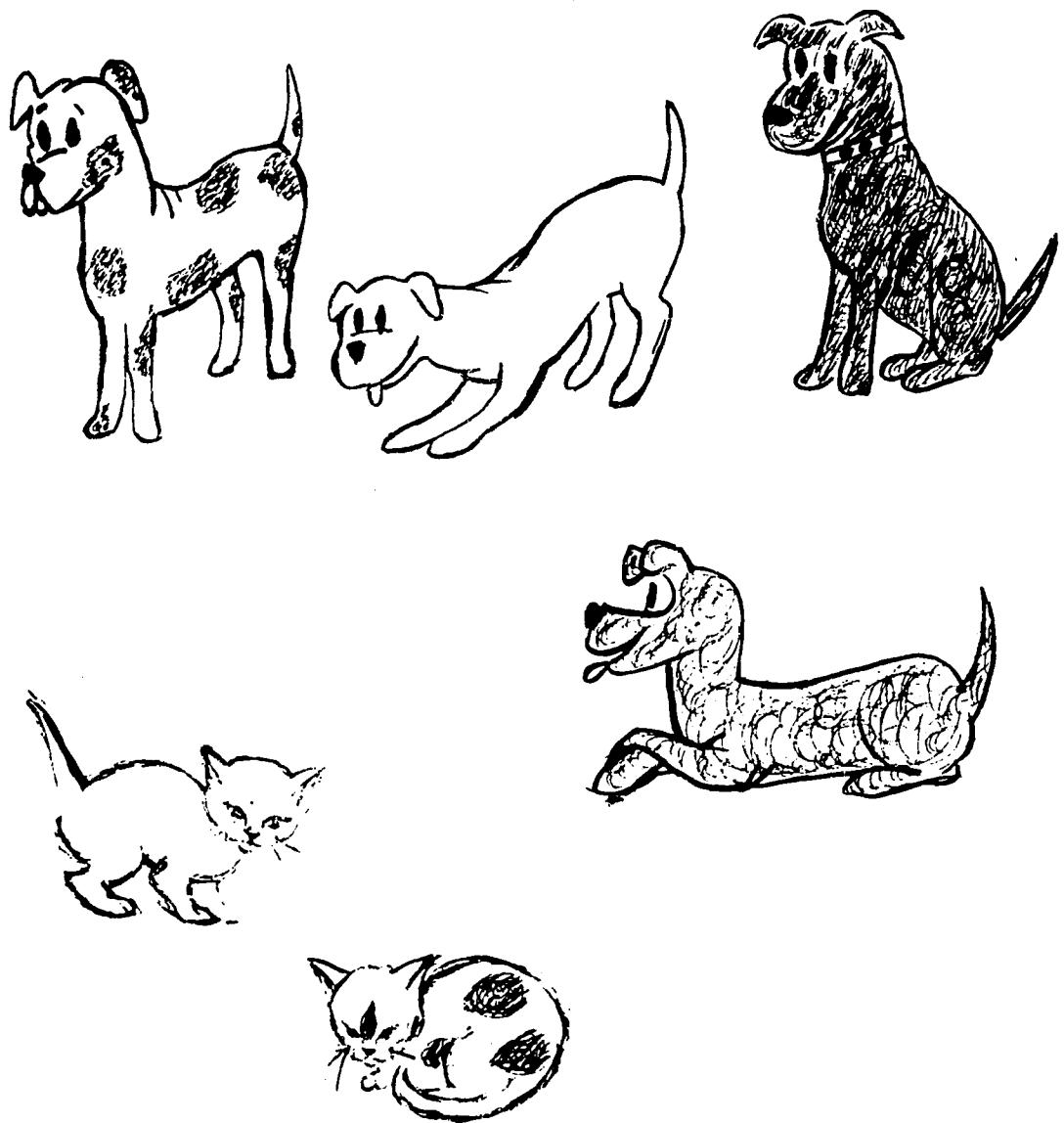
Worksheet 33a



How many members in this set of puppies?



How many kittens in this set of kittens?



How many members in the union of the set of puppies and the set of kittens ?

## Teacher Background on Sets Up

Elementary school programs commonly introduce addition by way of set union. Since this interpretation of addition is not satisfactory for irrational numbers (such as  $\sqrt{2}$  and  $\pi$ ), or even for fractional numbers, the Minnemast mathematics program bases addition on the number line. However, in order to help children appreciate the generality of addition, we want them to understand the relationship between addition and set union. Although we do not want to introduce the term addition until the number line, "Sets Up" provides some experience to prepare for addition.

Before children learn, for example, that 2 and 3 is 5, they should learn that adding 2 blocks to 3 blocks gives 5 blocks, and adding 2 crayons to 3 crayons gives 5 crayons; that is, they should learn that 2 of anything added to 3 of anything always produces a set having 5 members. It should also be realized that in order for the number of the union to be the sum of the numbers of the sets being united, the intersection of those sets must be the empty set.

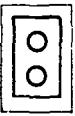
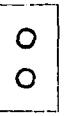
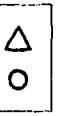
The purpose of "Sets Up" is to help children discover that the number of the union of two sets depends only on the numbers of the sets being united when the intersection of these two sets is the empty set; the number of the union does not depend in any other way on what the members of the sets happen to be. In seeking this discovery at this age level we do not expect sophisticated verbalization from the children.

While reviewing union, intersection, and counting, the activity also reviews circle and triangle.

## Sets Up

Before trying the activity with students, it is suggested that the teacher first play it with herself to learn how it goes.

Needed are a deck of cards as described below:

							
RED	1	1	1	1	2	2	8
BLUE	2	2	2	2	2	1	11
ORANGE	2	2	2	0	0	0	6
GREEN	2	1	1	0	0	0	4

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These cards can be selected from a pack of Minnemast Color Form Cards. The activity will be more interesting if this deck of 29 cards is shuffled before starting.

Search the deck for all the blue framed cards having 2 circles, and place them together. Next find the set of all blue framed cards having two triangles. How many members are there in the first set? In the second set?

To keep track of these numbers for further reference, make a chart of four columns as below:

Odd Numbered Set	Even Numbered Set	$\cup$	$\cap$
2	2		

Record the number of members in the first set in the column labeled "Odd Numbered Set", and record the number of members in the second set in the column for even numbered sets.

How many members in the union of these two sets? Place the appropriate numeral in the union (  $\cup$  ) column. How many members in the intersection? Place the appropriate numeral in the intersection (  $\cap$  ) column. If the intersection is the empty set, as in this example, put "0" in the intersection column.

Continue in the same manner through each of the following groups of sets (the first group is repeated here), recording the appropriate numerals in the chart each time.

Group A

1. All blue framed cards with 2 circles
2. All blue framed cards with 2 triangles

Group B

3. All orange framed cards with 2 circles
4. All orange framed cards with 2 triangles

Group C

5. All red framed cards with at least one circle
6. All red framed cards with at least one triangle

Group D

7. All plain cards with at least one blue circle
8. All plain cards with 2 blue triangles

Group E

9. All plain cards with two red circles
10. All plain cards with at least one red triangle

Group F

11. All green framed cards with at least one triangle
12. All green framed cards with at least one circle

Group G

13. All blue cards with both a triangle and a circle
14. All blue framed cards with both a triangle and a circle

Group H

15. All red framed cards
16. All orange framed cards with both a triangle and a circle

After completing the number chart for each group, study the patterns of numbers shown on the chart. Can you make any generalizations on the basis of this pattern of numbers?

The completed chart is shown below; check to be sure that you have counted and recorded accurately the number of members of each set, as well as those in the union and intersection.

Odd	Even,	Union	Intersection
2	2	4	0
2	2	4	0
2	2	3	1
3	2	5	0
2	3	5	0
3	2	4	1
3	2	3	2
3	2	5	0

"Sets Up" for the children is played in the same manner with only a few modifications. Familiarize the children with the cards, making sure to distinguish between the plain cards and the framed cards. Then distribute this same deck of 29 cards to the class, giving each child one card. If the class is smaller than 29 pupils, give two cards to some children. If the class is larger than 29 pupils, add as many orange or green plain cards as needed to provide each child with one card; do not add cards of any other kind.

Next, ask the children holding the cards described in Group A, Number 1 to stand. Ask for suggestions of ways of recording this set of children; elicit the suggestion of listing the names of the children in the set. Make a chalkboard chart, as below. Record the names of the children - members of each set, rather than the number of members.

		U	n
Mary, Johnny	Tommy, Suzy	Mary, Johnny, Tommy, Suzy	

If colored chalk is available, it may be helpful to label the sets with the design they represent. For example, for set 1 the teacher might write (in blue)  : Mary, Johnny; for set 10 (in red),  : Beth, George, Bill.

After recording the names of the children holding the cards described in Group A, 1 and 2, then ask all of the children in the union of these two sets to stand. Record their names in the appropriate column. Do the same for the intersection of these two sets.

Repeat this process, asking the children holding the cards described in each group, A - H, to stand and then record their names on the chart. The chart, when completed, will show the names of the children in each set, as well as the members of the union and intersection. The children may tend to confuse the terms "union" and "intersection" and a review of the terminology prior to the game may be profitable.

The next step is to add four columns to the chart. In these columns the number of members in the set will be recorded, rather than the names.

		U	∩
2	2	4	0
2	2	4	0
2	2	3	1

Ask the children to count the members of each set, then the number in the union and intersection. Record these numbers on the chart, as the children give them. If the children have difficulty in determining the number of members, it may be helpful to reconstruct the sets by having the children stand once again. If this is done, have the class count the children, rather than the names.

After the second chart is complete, ask the class to observe the patterns of numbers. If no one sees a pattern, the teacher (or children) might suggest other sets (not necessarily using the cards). Check each generalization to see if it works for all existing observations. Consider new sets and see if the generalizations will also work for them; that is, see if it has predictive value.

Suggested Activities on Penny, Nickel, Dime

1. Read the following story to introduce the coins.

Johnny, the Coin Detective

When Dad came home from work one evening, Johnny was waiting for him in the hall. Johnny seemed very upset and worried.

"What's the matter, Son," asked Dad.

"I'm worried about money, Daddy!"

"I'm afraid that is a worry for nearly everyone," laughed Dad. "Now, what bothers you about money?"

"Well, is big money always worth the most?"

"Not always. But usually it is. What coins are you worried about?"

"Dad, Ken knows pennies, and dimes, and nickels, and all I know is nickels."

"We'll have a money lesson tonight," said Dad.

So after supper that night, Johnny got his bank and emptied it on the living room rug. He and his Dad sat down in front of Johnny's collection of coins. There was the dime the fairy put under his pillow after he had pulled his first tooth. There were the pennies a neighbor had given him when he helped her clean out her garage. There were many other coins Johnny had collected during the past year.

"Come on," said Dad. "Let's see how many coins you know. We'll look at the tiniest one first. Dad took out a tiny silver coin.

"What is this?"

"I've always called it my silver penny when I play with my money." replied Johnny.

"This tiny silver coin is a dime. It is sometimes called a ten cent piece."

"How much is it worth, Dad? Does it take ten to make a penny?"

"No, Johnny. This is one place where the smaller coin is more valuable. It doesn't take ten dimes to make a penny. But it does take ten pennies to make a dime."

"Dad, I know about nickels. I know a nickel is worth an ice cream cone or five bubble gums," exclaimed Johnny.

"That's right, Johnny. But we measure coins by their value in cents. A nickel is worth five cents. Could you figure out how many nickels a dime is worth?" (Let children tell what they know about the relative value of these two coins. Emphasize the fact that it takes two nickels to be of equal value with a dime.)

Dad picked up three coins: a penny, a dime, and a nickel. He put them in front of Johnny. "Now, Johnny," he said, "I want you to arrange these in a row. Put the most valuable coin first; the next in value in the middle, and the one worth the least, last. Do you think you can do it?"

Johnny did it.

Could you?

2. Use flannelboard cut-outs or chalkboard drawings to illustrate the three coins. Label them 1¢, 5¢ and 10¢. Discuss which coin has the least monetary value and which has the most. Have the children point out or arrange the coins in order of value.
  
3. Discuss the money value of different coin groupings. Demonstrate that one dime is worth 10 pennies, 2 nickels or 1 nickel and 5 pennies.

4. Give each child a placard labeled 1¢, 5¢, or 10¢. Have them group themselves to form certain monetary values. For example, one 5¢ child is equal to five 1¢ children.
5. Give each child a set of play coins. These can be made of cardboard, as a class project. Offer certain things for sale; e.g. teacher's desk, 10¢. Have the children present different coin combinations as the purchase price.
6. Have the class play "store". Appoint committees to label the articles to be sold and to take turns being "storekeeper".